Chapter 5

Factors Affecting Bond Yields and the Term Structure of Interest Rates

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Base Interest Rate

- The minimum required interest rate by bond investors
- Benchmark interest rate
- A riskless return
- the yield of "on the run" (most recently issued) Treasury security
  - Backed by the full faith of U.S. government
  - Most liquid market

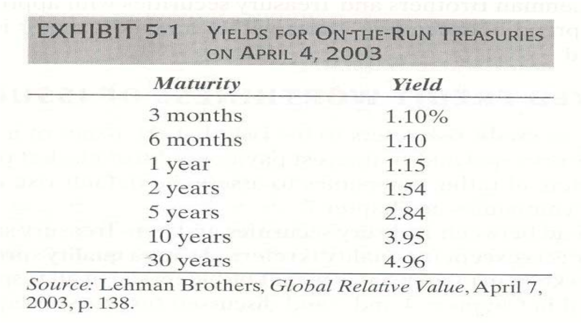


EXHIBIT 5-1 YIELDS FOR ON-THE-RUN TREASURIES ON APRIL 4, 2003

Maturity	*Yield*
3 months	1.10%
6 months	1.10
1 year	1.15
2 years	1.54
5 years	2.84
10 years	3.95
30 years	4.96

Source: Lehman Brothers, *Global Relative Value*, April 7, 2003, p. 138.

Benchmark Spreads

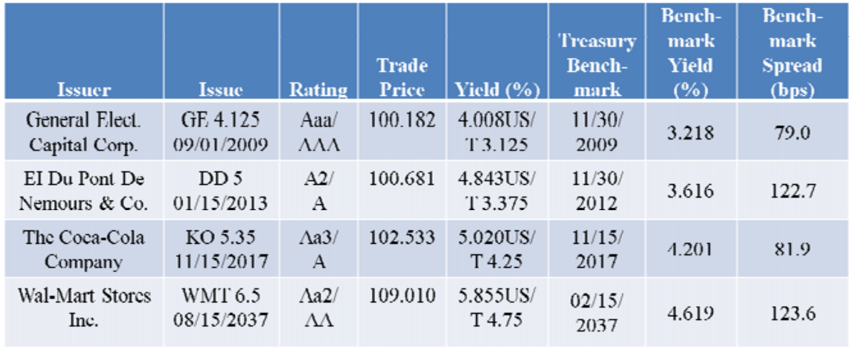
- The difference between the yields of any two bonds is called a *yield spread*

  yield spread = *yield on bond A – yield on bond B*
- *Benchmark spread=*

  yield on non-benchmark bond – yield on benchmark bond
  - compensation that the market is offering for bearing the risks associated with the non-benchmark bond that do not exist for the benchmark bond
  - treated as risk premium

Exhibit 5-2

U.S. Treasury Security Yields on December 28, 2007



Issuer	Issue	Rating	Trade Price	Yield (%)	Treasury Bench-mark	Bench-mark Yield (%)	Bench-mark Spread (bps)
General Elect. Capital Corp.	GE 4.125 09/01/2009	Aaa/ AAA	100.182	4.008US/ T 3.125	11/30/ 2009	3.218	79.0
EI Du Pont De Nemours & Co.	DD 5 01/15/2013	A2/ A	100.681	4.843US/ T 3.375	11/30/ 2012	3.616	122.7
The Coca-Cola Company	KO 5.35 11/15/2017	Aa3/ A	102.533	5.020US/ T 4.25	11/15/ 2017	4.201	81.9
Wal-Mart Stores Inc.	WMT 6.5 08/15/2037	Aa2/ AA	109.010	5.855US/ T 4.75	02/15/ 2037	4.619	123.6

Measuring Risk Premium on a Relative Basis

$$\textit{relative yield spread} = \frac{\textit{yield on bond A - yield on bond B}}{\textit{yield on bond B}}$$

$$\textit{yield ratio} = \frac{\textit{yield on bond A}}{\textit{yield on bond B}}$$

Factors that Affect the Yield Spread

- the type of issuer, the issuer’s perceived credit worthiness
- the term or maturity of the instrument
- provisions that grant either the issuer or the investor the option to do something
- the taxability of the interest received by investors
- the expected liquidity of the security

Types of issuers

- Market sectors
  - U.S government, U.S. government agencies, municipal governments, domestic/foreign corporations, foreign governments
- These classifications are referred to as *market sectors*.
- Different sectors have different abilities to satisfy the obligation
- Some market sectors are further subdivided into categories intended to reflect common economic characteristics.
  - Ex: industrial, utility, finance, and non-corporate
- intermarket sector spread
  - the yield difference due to type of issuers
  - Example: spread between Treasury securities and some other sector with the same maturity
- Intramarket sector spread
  - the yield between two issues within a market sector

Perceived credit worthiness of issuer

- Default risk / credit risk
  - the risk that bond issuers can’t make timely principal or interest payments
  - The higher the default risk, the higher the required yield
- Quality spread / credit spread
  - The spread between Treasury issues and non-Treasury issues that are identical in all other respects
- Default risk can be accessed via commercial rating companies
  - Fitch, S&P, Moody
- Examples of credit spreads on December 28, 2007, are provided in Exhibit 5-3

Exhibit 5-3

Corporate Bond Yields and Risk Premium Measures Relative to Treasury Yields on December 28, 2007

Maturity	Rating	Corporate Yield (%)	Treasury Yield (%)	Yield Spread (bps)
5-year	AAA	4.60	3.49	111
5-year	AA	4.91	3.49	142
10-year	AAA	5.03	4.07	96
10-year	AA	5.32	4.07	125

Note: Yields were reported on *finance. yahoo. com*. The original source is *ValuBond*.

Inclusion of options

- call provision
  - Benefit issuers by allowing them to replace an old bond issue with a lower-interest-cost issue
  - bad for bondholder because the bondholder must reinvest the proceeds at a lower interest rate
- Put provision, convertibility,…
- An analytical measure called the *option-adjusted spread* **(***OAS***)** is the yield spread after adjusting for the value of the embedded options.
- Market participants will require a larger (smaller) spread for bonds with options in favor to the issuers (bondholders)

Taxability of interest

- certain types of bond interest income are tax exempted
  - Interest income from municipal bonds is exempted from federal tax
- tax-exempt bonds provide lower yield than the comparable taxable bonds
  - After-tax yield

$$\textit{after - tax yield} = \textit{pretax yield} \times (1 - \textit{marginal tax rate})$$

$$= 0.05 \times (1 - 0.39) = 0.0305$$

  - Equivalent taxable yield
    - The yield that must be offered on a taxable bond to give the same after-tax yield as a tax-exempt issue

$$\textit{equivalent taxable yield} = \frac{\textit{tax exempt yield}}{1 - \textit{marginal tax rate}} = \frac{0.034}{1 - 0.39} = 0.0557$$

    - The higher the marginal tax rates, the higher the equivalent taxable yield

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Exhibit 5-4

AAA Municipal Yields versus AAA Corporate Yields

Maturity	Municipal Yield (%)	Corporate Yield (%)	Yield Ratio
5-year	3.35	4.91	0.68
10-year	3.87	5.03	0.77
20-year	4.59	5.32	0.86

Note: Yields were reported on finance. yahoo. com. The original source is ValuBond.

Because of the tax-exempt feature of municipal bonds, the yield on municipal bonds is less than Treasuries

Yield ratios are different due to tax asymmetry

- State and local governments may tax interest income on bond issues that are exempt from federal income taxes.
- Some municipalities' exempt interest income from all municipal issues from taxation; others do not.
- Some states exempt interest income from bonds issued by municipalities within the state but tax the interest income from bonds issued by municipalities outside the state.
- Municipalities are not permitted to tax the interest income from securities issued by the U.S. Treasury.
- Thus part of the spread between Treasury securities and taxable non-Treasury securities of the same maturity reflects the value of the exemption from state and local taxes.

Liquidity of the issues

- liquidity: the ability to immediately buy and sell bonds at fair prices
- the higher the liquidity, the lower the required yield
- On-the-run Treasury issues have greatest liquidity
- The liquidity risk premium may be large but is difficult to measure

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On-the-run bonds are most recently issued U.S. Treasury bonds of a particular maturity. Opposite to off-the-run bonds.

Financeability of an issue

- Some bonds can be used as collateral for borrowing funds
  - Repo (repurchase agreement) market
  - Bond dealers provides the funds by charging the repo rate
- A structure of repo rates depending on the maturity of the loan and the specific issue being financed.
  - When dealers are in need of particular issues to cover a short position
    - Such issues will have lower yield (higher price)
- A portion of the spread between on-the-run and off-the-run Treasury issues reflect the financing advantage of the on-the-run issues

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Term to maturity

- The time remaining on a bond's life
- Volatility of a bond depends on its term to maturity.
  - Given other thing equal, the longer the term to maturity of a bond, the greater the price volatility.
- Bonds are classified into three maturity sectors
  - Short term: 1-5 years
  - Intermediate term: 5-12 years
  - Long term: greater than 12 years
- Maturity spread
  - The spread between any two maturity sectors
  - longer-term bonds vs. shorter-term bonds
- Term structure of interest rates
  - The relationship between the yields on otherwise comparable securities with different maturities

Types of yield curve

- Yield curve
  - The graphical depiction of the yields of different maturity bonds of the same credit quality and type
  - shows the relationship between yields and maturity dates for a set of similar bonds, usually Treasuries, at a given point in time
- Types of yield curves:
  - Yield curve
    - Yield to maturity of Treasury issues
  - Spot rate curve
    - Yield of zero-coupon Treasury issues
  - Par coupon curve
    - The yield that makes the bond price equal to its par value

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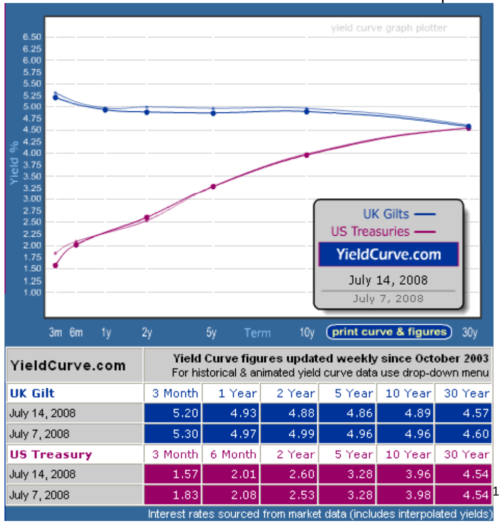
Types of yield curve

- UK and US yield to maturity

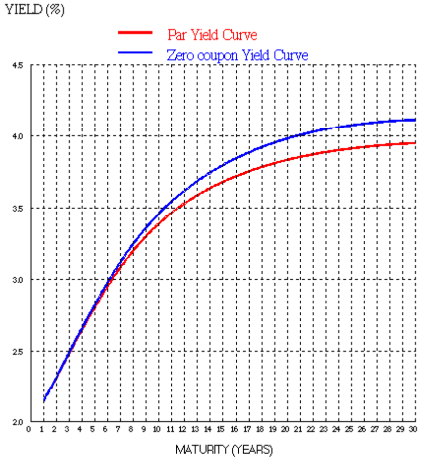


YieldCurve.com	Yield Curve figures updated weekly since October 2003 For historical & animated yield curve data use drop-down menu					
UK Gilt	3 Month	1 Year	2 Year	5 Year	10 Year	30 Year
July 14, 2008	5.20	4.93	4.88	4.86	4.89	4.57
July 7, 2008	5.30	4.97	4.99	4.96	4.96	4.60
US Treasury	3 Month	6 Month	2 Year	5 Year	10 Year	30 Year
July 14, 2008	1.57	2.01	2.60	3.28	3.96	4.54
July 7, 2008	1.83	2.08	2.53	3.28	3.98	4.54

Interest rates sourced from market data (includes interpolated yields)

Types of yield curve

- Euro yields on Aug., 5, 2005
  - Spot rate curve
  - Par yield curve

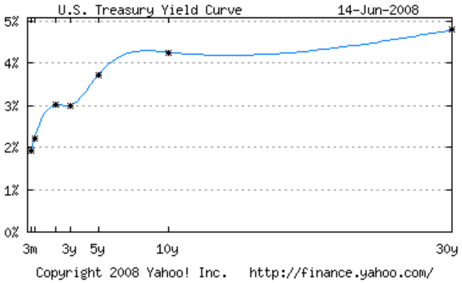


Why Treasury Securities are Usually Used to Draw Yield Curves?

- Treasury securities are used to draw the yield curve
  - For two reasons
    - Default free
    - Least problem of illiquidity or infrequent trading
  - Serve as a benchmark for pricing bonds
  - To set yields in all other sectors of the debt market

Yield curve example

- upward-slope yield curve indicates that the long-term bonds have higher yield

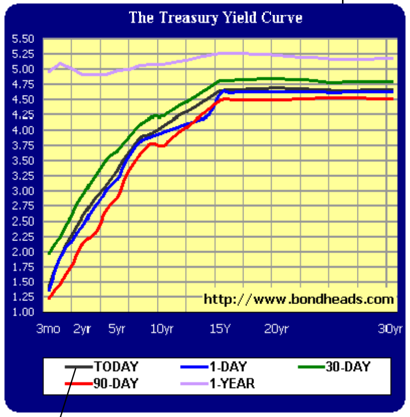


US Treasury Bonds

Maturity	Yield	Yesterday	Last Week	Last Month
3 Month	1.91	1.92	1.78	1.82
6 Month	2.22	2.21	1.91	1.89
2 Year	3.02	3.03	2.37	2.53
3 Year	3	3.01	2.34	2.54
5 Year	3.72	3.69	3.17	3.22
10 Year	4.25	4.21	3.91	3.92
30 Year	4.78	4.76	4.62	4.63

Yield curve example

- The shape of the yield curve changes over time
- The level of the yield curve changes as well
- Yields on Treasury bonds are considered risk-free and are thus a benchmark for determining the yield on other types of debt



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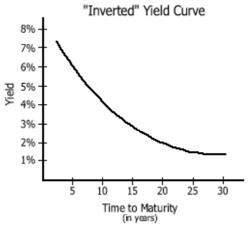
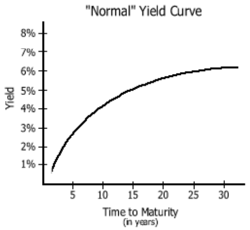
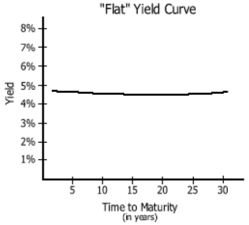
The shape of yield curve

- a normal yield curve indicates that investors require a higher rate of return for taking the added risk of lending money for a longer time
- a steeper normal curve indicates that investors expect strong future economic growth and higher future inflation (and thus higher interest rates)





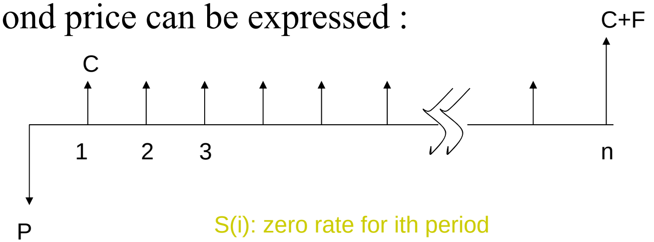


Why the yield curve should not be used to price a bond

- The bond pricing formula discounts all the cash flows with one rate
  - Can’t be applied to different cash flow patterns
- The cash flows of a bond can be viewed as a collection of a series of zero-coupon bonds
  - See next slide
- Each payment needs to be discounted using a yield (spot rate) associated with its maturity
- Yield to maturity is a complex average yield over the entire life of a Treasury bond
  - Not the zero-coupon yield of the maturity

Spot Rate (Zero rate)

- It is defined as yield of zero coupon bond
- A spot rate curve (zero-coupon yield curve) is a plot of spot rates against maturity.
- Bond price can be expressed :



S(i): zero rate for ith period

$$P = C\times(1+S(1))^{-1} + C\times(1+S(2))^{-2} + \ldots + (C+F)\times(1+S(n))^{-n}$$

y: bond yield

$$P = C\times(1+y)^{-1} + C\times(1+y)^{-2} + \ldots + C\times(1+y)^{-n} + F\times(1+y)^{-n}$$

Spot rate curve should be Constructed Theoretically

- Spot rate curve
  - The yield curve depicted by using a series of zero-coupon T-bonds
    - a zero-coupon T-bonds has single cash flow at maturity, therefore, appropriately measures the spot yield for investing funds for the particular horizon
  - The "real" term structure of interest rates
  - Problems associated with zero-coupon T-bonds
    - There are no zero-coupon Treasury debt issues with a maturity greater than one year
    - Derive from theoretical considerations as applied to the yields of the actually traded Treasury debt securities.
      - *theoretical spot rate curve*

Constructing the theoretical spot rate curve from Treasury bonds

- The Treasury issues that are candidates for inclusion are
  - on-the-run Treasury issues
  - on-the-run Treasury issues and selected off-the-run Treasury issues
  - all Treasury coupon securities, and bills
  - Treasury coupon strips

On-the-run Treasury issues

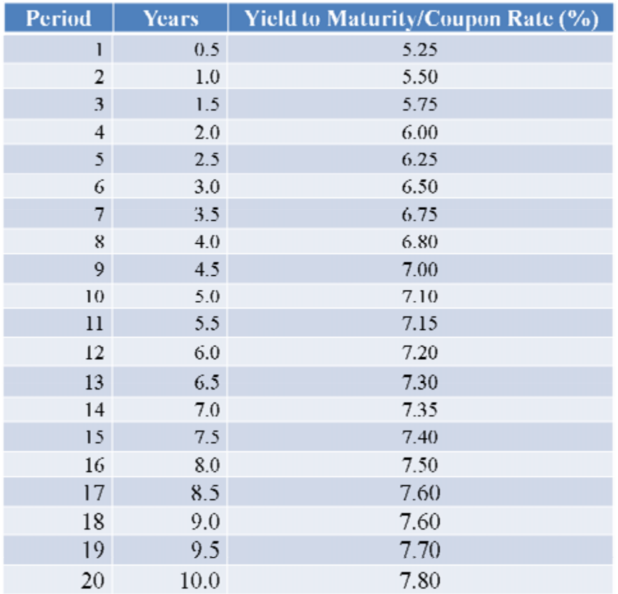
Generate Missing Yield

- These issues include 3-month, 6-month, and 1-year Treasury bills; 2-year, 5-year, and 10-year Treasury notes; and 30-year Treasury bond.
- Treasury bills are zero-coupon instruments; the notes and the bond are coupon securities.
- Providing the best liquidity and the most accurate yield to maturity
  - observed yield for each on the run issue
  - For the coupon issue→ Use par coupon yield
- Construct a theoretical yield curve with 60 semiannual spot rates: 6 month rate to 30-year rate. (Exclude 3-month)
  - 54 missing maturity points are estimated, says by linear interpolation

$$\frac{\textit{yield at higher maturity} - \textit{yield at lower maturity}}{\textit{number of semiannual periods between the two maturity points} + 1}$$

See next slide

Exhibit 5-6 Maturity and Yield to Maturity for 20 Hypothetical Treasury Securities



Period	Years	Yield to Maturity/Coupon Rate (%)
1	0.5	5.25
2	1.0	5.50
3	1.5	5.75
4	2.0	6.00
5	2.5	6.25
6	3.0	6.50
7	3.5	6.75
8	4.0	6.80
9	4.5	7.00
10	5.0	7.10
11	5.5	7.15
12	6.0	7.20
13	6.5	7.30
14	7.0	7.35
15	7.5	7.40
16	8.0	7.50
17	8.5	7.60
18	9.0	7.60
19	9.5	7.70
20	10.0	7.80

Ex:
1-year yield=5.5%
2-year=6%
(6%-5.5%)/2=0.25%

1.5y→5.5+0.25%

On-the-run Treasury issues

Problem:

- There is a large gap between some of the maturities points
  - No issues between 5- and 10-year maturity points and between 10- and 30-year maturity points
  - Yields of the on-the-run issues may understate the true yield due to their favorable financing opportunity in the repo market

→True yield is greater than the observed one.

Bootstrapping Method to Derive Theoretical Spot Rates

- Bootstrapping method
  - Suppose the YTM of three bonds are as below, the 0.5-year and 1-year bond is a zero-coupon bond, others are coupon bonds

	Maturity（year）	YTM	Coupon	Zero Rate
Zero bond	0.5	5.25%	0%	5.25%
	1	5.5%	0%	5.5%
	1.5	(Par yield)5.75%	5.75%	See next slide

Bootstrapping

- Assuming the spot rate for 0.5, 1, 1.5 years are Z_1, Z_2, and Z_3,
  - Z_1 = 5.25% (the YTM of 0.5-year zero coupon bond)
  - Z_2 =5.5% (the YTM of 1-year zero coupon bond)
  - Z_3 can be calculated using the pricing model of 1.5-year coupon bond （Par coupon yield= coupon rate = 5.75%）
  - Coupon=100*0.0575/2=2.875

$$100 = \frac{2.875}{(1+Z_1/2)} + \frac{2.875}{(1+Z_2/2)^2} + \frac{102.875}{(1+Z_3/2)^3} \Rightarrow Z_3/2 = 2.8798\%$$

Annualized rate=2.8798%*2 ≈ 5.76%

Obtaining the Zero Curve

- Z_1, Z_2, and Z_3 are the spot rate of zero coupon bond
- By repeating the above steps, we can obtain zero curves.
- Note that spot rate curve is different from the usual “yield curve” which typically depicts the market yield to maturity of coupon bonds
- See next slide for zero curve →Compare to Exhibit 5-6 for (par) yield curve

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Exhibit 5-7 Theoretical Spot Rates

Period	Years	Yield to Maturity/Coupon Rate (%)
1	0.5	5.25
2	1.0	5.50
3	1.5	5.76
4	2.0	6.02
5	2.5	6.28
6	3.0	6.55
7	3.5	6.82
8	4.0	6.87
9	4.5	7.09
10	5.0	7.20
11	5.5	7.26
12	6.0	7.31
13	6.5	7.43
14	7.0	7.48
15	7.5	7.54
16	8.0	7.67
17	8.5	7.80
18	9.0	7.79
19	9.5	7.93
20	10.0	8.07

On-the-run and selected off-the-run Treasury issues

- To fill some gaps of on-the-run issues
- Typically 20- and 25-year issues
- Linear intrapolating and bootstrapping

All Treasury Coupon Securities and Bills

- Using only on-the-run issues, even when extended to include a few off-the-run issues, may fail to recognize the information embodied in Treasury prices
- It might be more appropriate to use all Treasury coupon securities and bills to construct the theoretical spot rate curve.
  - not include callable bonds
- Use statistical method instead of bootstrapping
  - since there may be more than one yield for each maturity
  - Exponential spline fitting

Treasury Coupon Strips

- A bond can be resold into two parts
  - (1) a principal component
  - (2) the right to receive the coupon payments→coupon strips
- Strips are zero-coupon Treasury securities
- Problem by using observed rates on strips
  - the liquidity of the strips market is not as great as bond markets
  - the tax treatment of strips is different
  - non–U.S. investors find it advantageous to trade off yield for tax advantages associated with a strip
    - Ex: treat as capital gain at a favorable tax rate

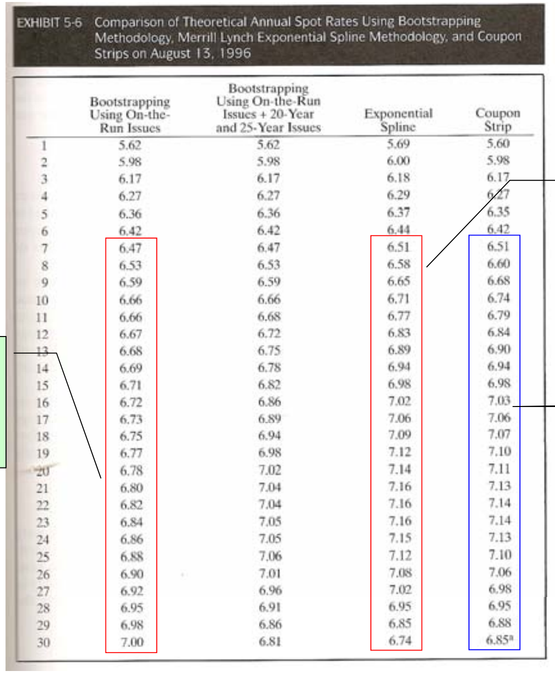


EXHIBIT 5-6 Comparison of Theoretical Annual Spot Rates Using Bootstrapping Methodology, Merrill Lynch Exponential Spline Methodology, and Coupon Strips on August 13, 1996

	Bootstrapping Using On-the-Run Issues	Bootstrapping Using On-the-Run Issues + 20-Year and 25-Year Issues	Exponential Spline	Coupon Strip
1	5.62	5.62	5.69	5.60
2	5.98	5.98	6.00	5.98
3	6.17	6.17	6.18	6.17
4	6.27	6.27	6.29	6.27
5	6.36	6.36	6.37	6.35
6	6.42	6.42	6.44	6.42
7	6.47	6.47	6.51	6.51
8	6.53	6.53	6.58	6.60
9	6.59	6.59	6.65	6.68
10	6.66	6.66	6.71	6.74
11	6.66	6.68	6.77	6.79
12	6.67	6.72	6.83	6.84
13	6.68	6.75	6.89	6.90
14	6.69	6.78	6.94	6.94
15	6.71	6.82	6.98	6.98
16	6.72	6.86	7.02	7.03
17	6.73	6.89	7.06	7.06
18	6.75	6.94	7.09	7.07
19	6.77	6.98	7.12	7.10
20	6.78	7.02	7.14	7.11
21	6.80	7.04	7.16	7.13
22	6.82	7.04	7.16	7.14
23	6.84	7.05	7.16	7.14
24	6.86	7.05	7.15	7.13
25	6.88	7.06	7.12	7.10
26	6.90	7.01	7.08	7.06
27	6.92	6.96	7.02	6.98
28	6.95	6.91	6.95	6.95
29	6.98	6.86	6.85	6.88
30	7.00	6.81	6.74	6.85[a]

Far away from the rate using coupon strip: Can't catch hump-shaped

Much closer to the rate using coupon strip

Hump-shaped

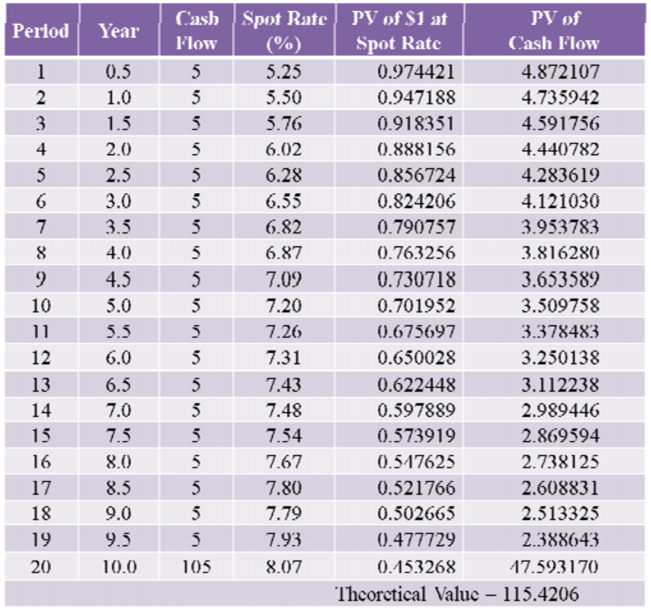
Using the theoretical spot rate curve

- Arbitrage forces a Treasury to be priced based on spot rates and not the yield curve.
  - The ability of traders to create value by stripping forces Treasuries to be priced based on the spot rates.
- Pricing bonds using the theoretical spot rates
  - Each cash flow is discounted at a different spot rate but not the same discount rate

$$P = \frac{C}{(1+sr_1)^1} + \frac{C}{(1+sr_2)^2} + \frac{C}{(1+sr_3)^3} + \cdots + \frac{C}{(1+sr_n)^n} + \frac{M}{(1+sr_n)^n}$$

  - Determining the value of a 10% 10-year T-bond
    - See next slide

Exhibit 5-8 Theoretical Spot Rates



Period	Year	Cash Flow	Spot Rate (%)	PV of $1 at Spot Rate	PV of Cash Flow
1	0.5	5	5.25	0.974421	4.872107
2	1.0	5	5.50	0.947188	4.735942
3	1.5	5	5.76	0.918351	4.591756
4	2.0	5	6.02	0.888156	4.440782
5	2.5	5	6.28	0.856724	4.283619
6	3.0	5	6.55	0.824206	4.121030
7	3.5	5	6.82	0.790757	3.953783
8	4.0	5	6.87	0.763256	3.816280
9	4.5	5	7.09	0.730718	3.653589
10	5.0	5	7.20	0.701952	3.509758
11	5.5	5	7.26	0.675697	3.378483
12	6.0	5	7.31	0.650028	3.250138
13	6.5	5	7.43	0.622448	3.112238
14	7.0	5	7.48	0.597889	2.989446
15	7.5	5	7.54	0.573919	2.869594
16	8.0	5	7.67	0.547625	2.738125
17	8.5	5	7.80	0.521766	2.608831
18	9.0	5	7.79	0.502665	2.513325
19	9.5	5	7.93	0.477729	2.388643
20	10.0	105	8.07	0.453268	47.593170
				Theoretical Value – 115.4206	

Spot rates and forward rates

- Spot rates
  - the required return rate (discount rate) for a cash payment in the future
  - the return for investing a fund beginning from now to another future time
  - example:
    - the yield of investing in a 3-year zero-coupon bond from now
  - the yield curve of spot rate is calculated from zero-coupon T-bill/bonds
- Forward rates
  - future interest rate
  - the return for investing a fund beginning from some future time to another future time
  - example:
    - the return of investing a bond 3 years later and holding for 2 years

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Forward Rates

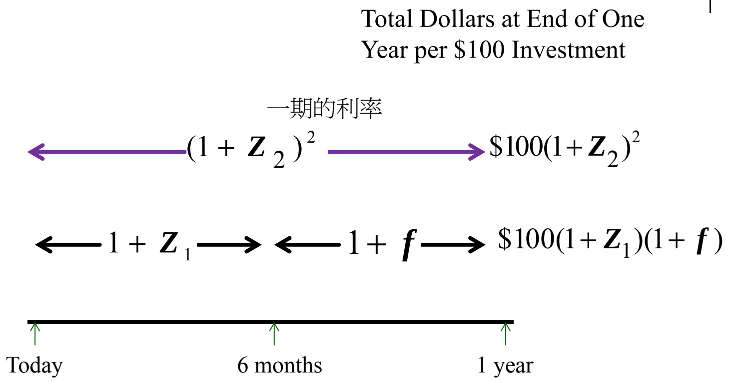
Consider the following two investment alternatives for an investor who has a one-year investment horizon.

1. *Alternative 1: Buy a one-year instrument.*
2. *Alternative 2: Buy a six-month instrument and when it matures in six months, buy another six-month instrument.*

- With *1*, the investor will realize the one-year spot rate with known repayment
- With *2*, the investor will realize the 6-month spot rate, but the 6-month rate 6 months from now is unknown.
- The rate that will be earned over one year is uncertain.

EXHIBIT 5-9

Two Alternative One-Year Investments



Total Dollars at End of One Year per $100 Investment

一期的利率

$(1+Z_2)^2$ $\$100(1+Z_2)^2$

$1+Z_1$ $1+f$ $\$100(1+Z_1)(1+f)$

Today

6 months

1 year

Two strategies should generate the same yield

- If any strategy yields higher return than the other, more investors will undertake the higher-yield strategy
- Price of the higher-yield strategy (bonds) will increase, which implies a decreasing of the yield
- At equilibrium, two strategies generate equal return
- The *f* that makes investors indifferent between the two alternative is forward rate

Calculating forward rates

- Forward rate can be calculated from two spot rates

$$(1+z_2)^2=(1+z_1)(1+f)$$

$$f=\frac{(1+z_2)^2}{1+z_1}-1$$

- Example:
  - One-year spot rate z_1=0.0525
  - Two-year spot rate z_2=0.055

$$f=\frac{(1.055)^2}{1.0525}-1=0.0575$$

    - The bond-equivalent basis is 5.75%

Calculating forward rates

- Finding a One-Year Forward Rate
  - Using term structure of interest rates from January 29, 1999, find the one-year forward rate for year three.

1-year Treasury bill	4.51%
2-year Treasury note	4.58%
3-year Treasury note	4.57%

$${}_2f_3=\left[\frac{(1+.0457)^3}{(1+.0458)^2}\right]-1=0.0455 \text{ or } 4.55\%$$

Implications of forward rate

- A natural question about forward rates is how well they do at predicting future interest rates.
- The forward rate may not realize
  - Actual future spot rate may be different from the forward rate
  - Prediction power can be explained by theories introduced later
- Forward rates are hedgeable rates
  - One can purchase interest rate derivatives (bond futures) to lock in the future return, thus avoid the uncertainty in the future spot rate

---

Locking in the Forward Rates

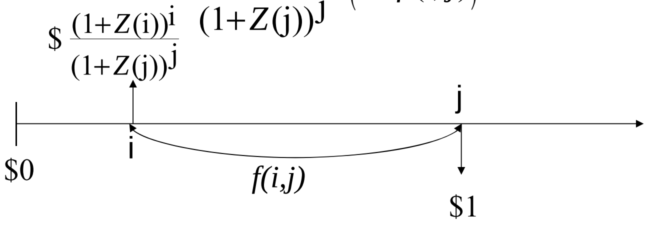
- Forward rates may not be realizes in the future (f(i,j) ≠Z(i,j)), but we can lock in any forward rate f(i,j).
- Now we can make following strategies.
  - Buy **1** unit *j-year* zero-coupon bond.
  - Sell $\frac{\mathbf{(1+Z(i))^i}}{\mathbf{(1+Z(j))^j}}$ units *i-year* zero-coupon bonds.
- No net initial investment, because

$$\frac{(1+Z(i))^i}{(1+Z(j))^j}\times\frac{1}{(1+Z(i))^i}-1\times\frac{1}{(1+Z(j))^j}=0$$

Locking in the Forward Rates

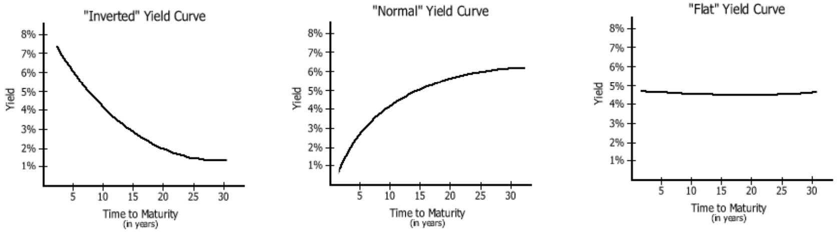
- At time *j* there will be a cash inflow of $**1**.
- At time *i* there will be a cash outflow of $ $\frac{(1+Z(i))^i}{(1+Z(j))^j}$
- The cash flow stream implies the rate *f(i,j)* between times *i* and *j*.

$$\frac{(1+Z(i))^i}{(1+Z(j))^j}\times\left(1+f(i,j)\right)^{j-i}=1$$



Determinants of the Shape of the Term Structure

- Three generic shapes in Exhibit 5-5



- Next slide shows five selective daily Treasury yield curves in tabular form.
- Two major theories have evolved to account for these observed shapes of the yield curve: *expectations theories* and *market segmentation theory*.

Exhibit 5-10 U.S. Treasury Yield Curve for Five Selective Dates

Day	3 mos	6 mos	1 yr	2 yrs	3 yrs	Shape
4/23/2001	4.38	4.78	5.06	5.84	5.73	Normal
4/10/1992	6.62	7.03	7.37	N/A	7.89	Steep
8/14/1981	15.34	15.04	NA	14.74	13.95	Inverted
1/3/1990	7.92	8.04	7.99	N/A	8.04	Flat
1/4/2001	4.82	5.07	5.03	5.56	5.44	Humped

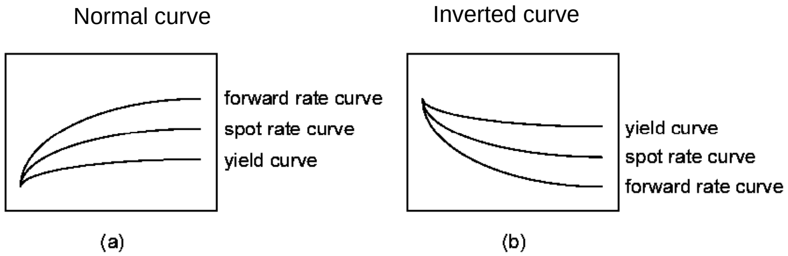
Theories on Term Structure

- There are several forms of the ***expectations theory***: *pure expectations theory*, *liquidity theory*, and *preferred habitat theory*.
  - share a hypothesis about the behavior of short-term forward rates
  - assume that the forward rates in current long-term bonds are closely related to the market's expectations about future short-term rates.
  - Theories differ as to whether other factors also affect forward rates
- *Pure expectations theory* postulates that no systematic factors other than expected future short-term rates affect forward rates;
- The *liquidity theory* and the *preferred habitat theory* assert that there are other factors.
  - also called *biased expectations theories*.

Pure expectation theory

- The forward rate exclusively represent the expected future rates
- The term structure at a given time reflects the market's current expectations of the family of future short-term rates
- A rising (declining) term structure indicates that the market expects short-term rates to rise (drop) throughout the relevant future
  - See the proofs in next two slides

零息利率曲線，遠期利率曲線關係



$$(1+Z_b)^b = (1+Z_a)^a(1+f(a,b))^{b-a}$$

當Zb>Za=> f(a,b)>Zb>Za

- Forward rate equals the average future spot rate,

$$f(a, b) = E[\ S(a, b)\ ]$$

- Implies that a normal spot rate curve is due to the fact that the market expects the future spot rate to rise.
  - $f(j, j+1) > S(j+1)$ if and only if $S(j+1) > S(j)$.
- Therefore, $E[\ S(j, j+1)\ ] > S(j+1) > \cdots > S(1)$ if and only if $S(j+1) > \cdots > S(1)$.
- Conversely, the spot rate is expected to fall if and only if the spot rate curve is inverted.

Liquidity theory

- The forward rates should reflect both interest-rate expectation and a "liquidity" (risk) premium
- Investors require higher yield for holding long-term bonds because such bonds are riskier
- Even if there are no expectation of future yield increasing, the yield curve is still upward because the long-term rates embody a liquidity premium

Preferred habitat theory

- The expectation of the future path of interest rates as well as risk premium
- In addition, borrowers and lenders have preferred maturity range
  - Institutions have holding periods dictated by the nature of their liabilities
- The Preferred Habitat Theory allows market participants to trade outside of their preferred maturity if adequately compensated for the additional risk
- Demand and supply of funds does not match for a given maturity range, some participants will shift to maturities showing the opposite imbalances
  - Participants are compensated by an appropriate risk premium whose magnitude will reflect the extent of aversion to either price or reinvestment risk

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Market segmentation theory

- Market participants have strong preferences for securities of particular maturity and buy and sell securities consistent with their maturity preferences
- Neither investors nor borrowers are willing to shift from one maturity sector to another
  - Differ from the preferred habitat theory
- The shape of the yield curve is determined by the supply and demand for securities within each maturity sector

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The Main Influences of the Shape of the Yield Curve

- Empirical evidence suggests that the three main influences on the shape of the Treasury yield curve are:
  i. the market's expectations of future rate changes
  ii. bond risk premiums (expected return difference due to different maturies.)
  iii. convexity bias.
- The convexity bias influence is the least well known of the three influences.
- The longer the maturity, the more convexity the security has.
  - Longer-term Treasury securities is more attractive due to convexity than shorter-term Treasury securities.
  - Investors are willing to pay more for longer-term Treasury securities and therefore accept lower returns.
- This influence on the shape of the Treasury yield curve is what is referred to as the convexity bias.

Swap Rate Yield Curve

- Interest rate swap market in most countries is increasingly used as an interest rate benchmark despite the existence of a liquid government bond market.
- The parties exchange interest rate payments on specified dates:
  - one party pays a fixed rate →called the *swap rate*
  - the other party pays a floating rate
- The floating rate is based on a reference rate, says the London Interbank Offered Rate (*LIBOR*).
- For Euro interest rate swaps, the reference rate is the Euro Interbank Offered Rate (Euribor)
- The relationship between the swap rate and maturity of a swap is called the *swap rate yield curve* or, more commonly, the *swap curve.*
- Because the reference rate is typically LIBOR, the swap curve is also called the *LIBOR curve*.

An Example of Interest rate Swaps

- An agreement by Microsoft to receive 6-month LIBOR & pay a fixed rate of 5% per annum every 6 months for 3 years on a notional principal of $100 million

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Cash Flows to Microsoft

		---------Millions of Dollars---------		
	LIBOR	*FLOATING*	*FIXED*	Net
Date	Rate	Cash Flow	Cash Flow	Cash Flow
Mar.5, 2004	4.2%			
Sept. 5, 2004	4.8%	+2.10	–2.50	–0.40
Mar.5, 2005	5.3%	+2.40	–2.50	–0.10
Sept. 5, 2005	5.5%	+2.65	–2.50	+0.15
Mar.5, 2006	5.6%	+2.75	–2.50	+0.25
Sept. 5, 2006	5.9%	+2.80	–2.50	+0.30
Mar.5, 2007	6.4%	+2.95	–2.50	+0.45

Exhibit 5-13

U.S., Euro, and U.K. Swap Rate Curve on October 16, 2007

Years to Maturity	Swap Rate (%) for:		
	U.S.	Euro	U.K.
1	4.96	4.72	6.19
2	4.85	4.65	5.97
3	4.90	4.64	5.91
5	5.12	4.65	5.80
7	5.18	4.69	5.70
10	5.33	4.77	5.56
12	5.39	4.82	5.50
15	5.46	4.88	5.40
20	5.52	4.92	5.27
15	5.54	4.92	5.14
30	5.54	4.90	5.04

Note: *The rates in this table were obtained from CLP Structured Finance's website, www.swap-rates.com.*

Swap Rate Yield Curve (continued)

- The swap curve is used as a benchmark in many countries outside the U.S.
- Unlike a country's government bond yield curve, the swap curve is not a default-free yield curve but reflects the credit risk of the counterparty to an interest rate swap.
- One would expect that if a country has a government bond market, the yields in that market would be the best benchmark; however, that is not necessarily the case.

Advantages of Using Swap Curves

i. In government bond market, some of the interest rates may not be representative of the true interest rate but instead be biased by some technical or regulatory factor

ii. To create a representative government bond yield curve, a large number of maturities must be available.

iii. The ability to compare government yields across countries is difficult because there are differences in the credit risk for every country.

Using Swap Rates to Bootstrap the LIBOR/Swap Zero Curve

- Consider a new swap where the fixed rate is the swap rate
- When principals are added to both sides on the final payment date the swap is the exchange of a fixed rate bond for a floating rate bond
- The floating-rate rate bond is worth par. The swap is worth zero. The fixed-rate bond must therefore also be worth par
- This shows that swap rates define par yield bonds that can be used to bootstrap the LIBOR (or LIBOR/swap) zero curve

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假設F= swap rate, P=par,

s_i=i-th period (year) spot rate,

N=# of swap periods,

$$P*F*e^{-s_1*1}+P*F*e^{-s_2*2}+...+P(1+F)e^{-s_N*N}=P$$