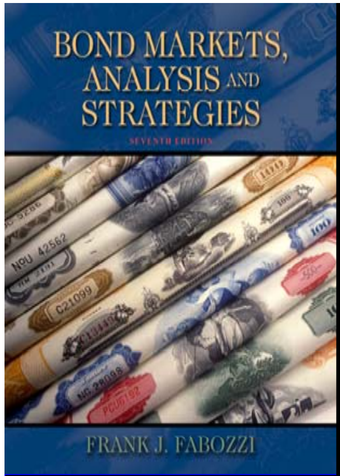


Chapter 4

Bond Price Volatility

Learning Objectives

After reading this chapter, you will understand

- ❖Focus on option-free bond
  - ❖the price-yield relationship
  - ❖ the price-volatility properties
- ❖ Duration:
  - –the Macaulay duration, modified duration, and dollar duration of a bond
  - ❖ a measure of a bond's price sensitivity to yield changes
  - ❖the spread duration measure for fixed-rate and floating-rate bonds
  - ❖portfolio duration
  - ❖ limitations of using duration as a measure of price volatility

Learning Objectives (continued)

After reading this chapter, you will understand

- ❖Convexity
  - ❖how price change estimated by duration can be adjusted for a bond's convexity
- ❖ how to approximate the duration and convexity of a bond
- ❖ the duration of an inverse floater
- ❖ how to measure a portfolio's sensitivity to a nonparallel shift in interest rates (key rate duration and yield curve reshaping duration)

Review of the Price-Yield Relationship for Option-Free Bonds

- ➢ An *increase* in the required yield *decreases* the PV of its expected cash flows and the bond's price.
- ➢ An *decrease* in the required yield *increases* the PV of its expected cash flows and the bond's price.
- ➢ See next slide:
- ➢ The percentage price change w.r.t. change in yield is not the same for all bonds.

Exhibit 4-1	Price–Yield Relationship for Six Hypothetical Bonds					
Required Yield (%)	Price at Required Yield (coupon/maturity in years)					
	9% / 5	9% / 25	6% / 5	6% / 25	0% / 5	0% / 25
6.00	112.7953	138.5946	100.0000	100.0000	74.4094	22.8107
7.00	108.3166	123.4556	95.8417	88.2722	70.8919	17.9053
8.00	104.0554	110.7410	91.8891	78.5178	67.5564	14.0713
8.50	102.0027	105.1482	89.9864	74.2587	65.9537	12.4795
8.90	100.3966	100.9961	88.4983	71.1105	64.7017	11.3391
8.99	100.0395	100.0988	88.1676	70.4318	64.4236	11.0975
9.00	100.0000	100.0000	88.1309	70.3570	64.3928	11.0710
9.01	99.9604	99.9013	88.0943	70.2824	64.3620	11.0445
9.10	99.6053	99.0199	87.7654	69.6164	64.0855	10.8093
9.50	98.0459	95.2539	86.3214	66.7773	62.8723	9.8242
10.00	96.1391	90.8720	84.5565	63.4881	61.3913	8.7204
11.00	92.4624	83.0685	81.1559	57.6712	58.5431	6.8767
12.00	88.9599	76.3572	77.9197	52.7144	55.8395	5.4288

Principles

- Bond prices move inversely with interest rates
- The relationship is not linear
- The shape is referred to as a *convex relationship*
  - The percentage price change is not the same for all bonds for a given change in yield
  - For very small changes in the yield, the percentage price change for a given bond is roughly the same for increased or decreased yield
  - For a given large change in basis point, the percentage price increase is greater than the percentage price decrease

    Can be numerically verified in next slide.

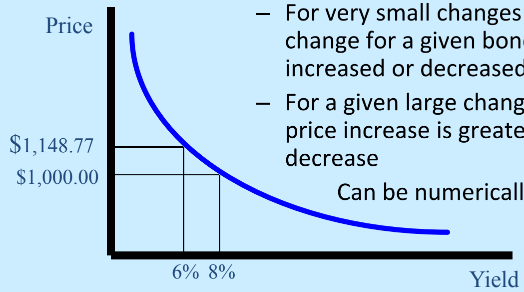


EXHIBIT 4-3	Instantaneous Percentage Price Change for Six Hypothetical Bonds						
Six hypothetical bonds, priced initially to yield 9%:							
9% coupon, 5 years to maturity, price = 100.0000 9% coupon, 25 years to maturity, price = 100.000 6% coupon, 5 years to maturity, price = 88.1309				6% coupon, 25 years to maturity, price = 70.3570 0% coupon, 5 years to maturity, price = 64.3928 0% coupon, 25 years to maturity, price = 11.0710			
Yield (%) Change to:	Change in Basis Points	Percentage Price Change (coupon/maturity in years)					
		9% / 5	9% / 25	6% / 5	6% / 25	0% / 5	0% / 25
6.00	-300	12.80	38.59	13.47	42.13	15.56	106.04
7.00	-200	8.32	23.46	8.75	25.46	10.09	61.73
8.00	-100	4.06	10.74	4.26	11.60	4.91	27.10
8.50	-50	2.00	5.15	2.11	5.55	2.42	12.72
8.90	-10	0.40	1.00	0.42	1.07	0.48	2.42
8.99	small -1	0.04	0.10	0.04	0.11	0.05	0.24
9.01	change 1	-0.04	-0.10	-0.04	-0.11	-0.05	-0.24
9.10	10	-0.39	-0.98	-0.41	-1.05	-0.48	-2.36
9.50	50	-1.95	-4.75	-2.05	-5.09	-2.36	-11.26
10.00	100	-3.86	-9.13	-4.06	-9.76	-4.66	-21.23
11.00	200	-7.54	-16.93	-7.91	-18.03	-9.08	-37.89
12.00	300	-11.04	-23.64	-11.59	-25.08	-13.28	-50.96

Characteristics of a Bond that Affect its Price Volatility

There are *two characteristics* of an option-free bond that determine its price volatility: ***coupon*** and ***term to maturity***.

1) First, for a given term to maturity and initial yield, the price volatility of a bond is greater, the lower the coupon rate.

✓ This characteristic can be seen by comparing the 9%, 6%, and zero-coupon bonds with the same maturity in the previous slide

2) Second, for a given coupon rate and initial yield, the longer the term to maturity, the greater the price volatility.

✓ This can be seen by comparing the five-year bonds with the 25-year bonds with the same coupon in the previous slide.

Effects of Yield to Maturity

- ➢ Holding other factors constant, the higher the yield to maturity at which a bond trades, the lower the price volatility..
- ✓ An implication of this is that for a given change in yields, price change is greater (lower) when yield levels in the market are low (high).

EXHIBIT 4-4	Price Change for a 100-Basis-Point Change in Yield for a 9% 25-Year Bond Trading at Different Yield Levels			
Yield Level (%)	Initial Price	New Price [a]	Price Decline	Percent Decline
7	$123.46	$110.74	$12.72	10.30
8	110.74	100.00	10.74	9.70
9	100.00	90.87	9.13	9.13
10	90.87	83.07	7.80	8.58
11	83.07	76.36	6.71	8.08
12	76.36	70.55	5.81	7.61
13	70.55	65.50	5.05	7.16
14	65.50	61.08	4.42	6.75

[a] *As a result of a 100-basis-point increase in yield.*

Measures of Bond Price Volatility

- ❖ Money managers, arbitrageurs, and traders need to have a way to measure a bond's price volatility to implement hedging and trading strategies.
- ❖ Three measures that are commonly employed:

1) price value of a basis point
2) yield value of a price change
3) duration

Price Value of a Basis Point

- ➢ Also called the dollar value of an 01,
- ➢ is the change of the bond price if the required yield changes by 1 bp.
- ➢ indicates dollar price volatility
  - ➢ opposed to percentage price volatility (price change as a percent of the initial price).
  - ➢ dividing the price value of a basis point by the initial price gives the percentage price change for a 1-basis-point change in yield.
- ➢ Typically, the price value of a basis point is expressed as the absolute value of the change in price.

Bond	Initial Price	Price at 9.01%	Price Value of a BP
5year 9%	100	99.9604	0.0396
25year 9%	100	99.9031	0.0987

Yield Value of a Price Change

- change in the yield to maturity for a specified price change
- compare the yield before and after a specified bond price changes

➢ The smaller this value, the greater the dollar price volatility, because it would take a smaller change in yield to produce a price change of *X* dollars.

Bond	Initial Price Minus a 32nd[a]	Yield at New Price	Initial Yield	Yield Value of a 32nd
5-year 9% coupon	99.96875	9.008	9.000	0.008
25-year 9% coupon	99.96875	9.003	9.000	0.003

[a] Initial price of 100 minus 1/32 of 1%.

Measures of bond price volatility

- Duration
  - the percentage price change for a given yield changes
  - a summary measure of bond price volatility
  - incorporate the effect of coupon and maturity

Macaulay Duration

- A measure of bond sensitivity to changes in interest rate
- The price of bond

$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \cdots + \frac{C}{(1+y)^n} + \frac{M}{(1+y)^n}$$

- The approximate change in price for a small change in yield, by taking the first derivative with respect to the required yield

$$\frac{dP}{dy} = \frac{(-1)C}{(1+y)^2} + \frac{(-2)C}{(1+y)^3} + \cdots + \frac{(-n)C}{(1+y)^{n+1}} + \frac{(-n)M}{(1+y)^{n+1}}$$

$$\frac{dP}{dy} = \frac{-1}{(1+y)}\left[\frac{1C}{(1+y)^1} + \frac{2C}{(1+y)^2} + \cdots + \frac{nC}{(1+y)^n} + \frac{nM}{(1+y)^n}\right]$$

- The approximate percentage change

$$\frac{dP}{dy}\frac{1}{P} = \frac{-1}{(1+y)}\left[\frac{1C}{(1+y)^1} + \frac{2C}{(1+y)^2} + \cdots + \frac{nC}{(1+y)^n} + \frac{nM}{(1+y)^n}\right]\frac{1}{P}$$

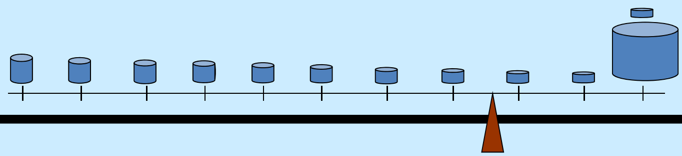
Macaulay duration

Macaulay Duration

- Macaulay's duration

$$\text{Macaulay duration} = \frac{\frac{1C}{(1+y)^1} + \frac{2C}{(1+y)^2} + \cdots + \frac{nC}{(1+y)^n} + \frac{nM}{(1+y)^n}}{P}$$

  - the average period of payment of a bond
  - weighted average term to maturity of the cash flows divided by prices, where the weights are the present value of the cash flow



Modified Duration

➢ Investors refer to the ratio of Macaulay duration to $1 + y$ as the modified duration. The equation is:

$$\text{modified duration} = \frac{\textbf{Macaulay duration}}{1+\boldsymbol{y}}$$

➢ The modified duration is related to the approximate percentage change in price for a given change in yield as given by:

$$\frac{\boldsymbol{dP}}{\boldsymbol{dy}}\frac{1}{\boldsymbol{P}} = -\text{modified duration}$$

where **dP** = *change in price*, **dy** = *change in yield*, **P** = *price of the bond.*

Measures of Bond Price Volatility

- Because for all option-free bonds modified duration is positive, $(dP/dy)(1/P)<0$,
- an inverse relationship between the approximate percentage price change and the yield change.
- This is to be expected from the fundamental principle that bond prices move in the opposite direction of the change in interest rates.
- Example for calculating Macaulay duration and modified duration of two five-year coupon bonds.
  - ✓ The durations computed in these exhibits are in terms of duration per period.

Duration (Conversion)

- In general, if the cash flows occur m times per year, the durations are adjusted by dividing by m, that is,

$$duration\ in\ years = \frac{\textbf{duration in } \boldsymbol{m} \textbf{ periods per year}}{\boldsymbol{m}}$$

Calculation of Duration

EXHIBIT 4-6 Calculation of Macaulay Duration and Modified Duration for 5-Year 6% Bond Selling to Yield 9%

Coupon rate: 6.00%
Term (years): 5
Initial yield: 9.00%

Period, t	Cash Flow[a]	PV of $1 at 4.5%	PV of CF	$t \times$ PVCF[b]
1	$ 3.00	0.956937	2.870813	2.87081
2	3.00	0.915729	2.747190	5.49437
3	3.00	0.876296	2.628890	7.88666
4	3.00	0.838561	2.515684	10.06273
5	3.00	0.802451	2.407353	12.03676
6	3.00	0.767895	2.303687	13.82212
7	3.00	0.734828	2.204485	15.43139
8	3.00	0.703185	2.109555	16.87644
9	3.00	0.672904	2.018713	18.16841
10	103.00	0.643927	66.324551	663.24551
Total			88.130923	765.89520

[a]Cash flow per $100 of par value.

$$\text{Macaulay duration (in half years)} = \frac{765.89520}{88.130923} = 8.69$$

$$\text{Macaulay duration (in years)} = \frac{8.69}{2} = 4.35$$

$$\text{Macaulay duration} = \frac{4.35}{1.0450} = 4.16$$

[b]Values are rounded.

Properties of Duration

- For coupon-bearing bonds, both Macaulay and modified durations are always less than term to maturity
- For zero-coupon bonds, Macaulay Duration is exactly the same as term to maturity, Modified is less than the maturity
- A longer term to maturity increases duration, all other things being equal
  - Duration increases with term to maturity at a decreasing rate

Bond	Macaulay Duration (years)	Modified Duration
9%/5-year	4.13	3.96
9%/25-year	10.33	9.88
6%/5-year	4.35	4.16
6%/25-year	11.10	10.62
0%/5-year	5.00	4.78
0%/25-year	25.00	23.92

Properties of Duration

- Lower coupon rates generally lead to longer durations, all other things being equal
- Higher yields lead to shorter durations

Yield (%)	Modified Duration
7	11.21
8	10.53
9	9.88
10	9.27
11	8.70
12	8.16
13	7.66
14	7.21

Approximating the Percentage Price Change

➢ The below equation can be used to approximate the percentage price change for a given change in required yield:

$$\frac{dP}{P} = -(modified\ duration)(dy)$$

Approximating the Dollar Price Change

➢ Modified duration is a proxy for the percentage change in price. Investors also like to know the dollar price volatility of a bond.

➢ For small changes in the required yield, the price change is estimated as

$$dP = -(dollar\ duration)(dy)$$

where Dollar duration= -(modified duration)*P

Ex: 6% 25-year bond sell at 70.357 given yield=9%. Dollar duration=747.2009

An increment of 1bp:

*dP=-747.2009*0.0001=-0.0747*

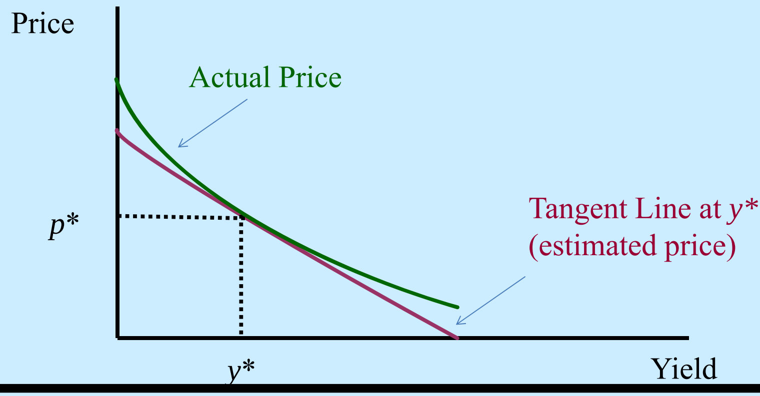
Bond price decreases is about -0.0747 →70.2824-70.357=-0.0746

Required Yield (%)	Price at Required Yield (coupon/maturity in years)					
	9% / 5	9% / 25	6% / 5	6% / 25	0% / 5	0% / 25
9.00	100.0000	100.0000	88.1309	70.3570	64.3928	11.0710
9.01	99.9604	99.9013	88.0943	70.2824	64.3620	11.0445

➢ Due to convex property between yield and price, dollar and modified durations are not adequate to approximate when dy is large.

➢ Duration will overestimate the price change when the required yield rises, thereby underestimating the new price.

➢ When the required yield falls, duration will underestimate the price change and thereby underestimate the new price.



Spread Duration

- This measure is used in two ways: for fixed bonds and floating-rate bonds.
- Duration measures Δbond value w.r.t. Δyield
- For fixed rate security:
  - Treasury bond → treasury rate
  - non-Treasury bond → treasury rate +credit spread
  - A measure of how non-Treasury bond's price change w.r.t. spread change is called
    - Spread duration
- For floating rate security:
  - Coupon reset as : reference rate+ quoted margin
  - Spread duration measures the change of security price w.r.t. the change in quoted margin.

Portfolio duration

- Portfolio duration is the weighted average duration of the bonds in the portfolio
- Example

Bond	Market Value	Portfolio Weight	Duration
A	$10 million	0.10	4
B	$40 million	0.40	7
C	$30 million	0.30	6
D	$20 million	0.20	2

  - Portfolio duration=0.1×4+0.4×7+0.3×6+0.2×2=5.4
- This linear property is only an approximation when the yield curve is not flat

Contribution to Portfolio duration

- Contribution to portfolio duration
  - Weight of issue in portfolio × duration of issue
  - Important to bond fund managers

Bond	Market Value	Weight in Portfolio	Duration	Contribution to Duration
A	$ 10,000,000	0.10	4	0.40
B	$ 40,000,000	0.40	7	2.80
C	$ 30,000,000	0.30	6	1.80
D	$ 20,000,000	0.20	2	0.40
Total	$100,000,000	1.00		5.40

Ex: Portfolios of Lehman Bother Portfolio Duration

- The portfolio duration is divided into two durations.
  - ✓ The first is the duration of the portfolio with respect to changes in the level of Treasury rates.
  - ✓ The second is the spread duration.
- ➢ Exhibit 4-7 (next slide) denotes a portfolio allocation on six sectors suggested by Lehman Bother
- ➢ Exhibit 4-8 shows the size of each sector in the Lehman Brothers U.S. Aggregate Index.

EXHIBIT 4-7	Calculation of Duration and Contribution to Portfolio Duration for an Asset Allocation to Sectors of the Lehman Brothers U.S. Aggregate Index: October 26, 2007		
Sector	**Portfolio Weight**	**Sector Duration**	**Contribution to Portfolio Duration**
Treasury	0.000	4.95	0.00
Agency	0.121	3.44	0.42
Mortgages	0.449	3.58	1.61
Commercial Mortgage-Backed Securities	0.139	5.04	0.70
Asset-Backed Securities	0.017	3.16	0.05
Credit	0.274	6.35	1.74
Total	1.000		4.52

EXHIBIT 4-8	Calculation of Duration and Contribution to the Lehman Brothers Aggregate Index Duration: October 26, 2007		
Sector	**Weight in Index**	**Sector Duration**	**Contribution to Index Duration**
Treasury	0.230	4.95	1.14
Agency	0.105	3.44	0.36
Mortgages	0.381	3.58	1.36
Commercial Mortgage-Backed Securities	0.056	5.04	0.28
Asset-Backed Securities	0.010	3.16	0.03
Credit	0.219	6.35	1.39
Total	1.000		4.56

The portfolio durations are close.

Ex: Portfolios of Lehman Bother Spread Duration

- The spread durations are in
  - Exhibit 4-9 (*see next slide*) and
  - Exhibit 4-10
- While the portfolio and the index have the same duration, the spread duration for the recommended portfolio is 4.60 vs. 3.49 for the index.
- The larger spread duration for the recommended portfolio is expected given the greater allocation to non-Treasury sectors.

EXHIBIT 4-9	Calculation of Spread Duration and Contribution to Portfolio Spread Duration for an Asset Allocation to Sectors of the Lehman Brothers U.S. Aggregate Index: October 26, 2007		
Sector	**Portfolio Weight**	**Sector Spread Duration**	**Contribution to Portfolio Spread Duration**
Treasury	0.000	0.00	0.00
Agency	0.121	3.53	0.43
Mortgages	0.449	3.62	1.63
Commercial Mortgage-Backed Securities	0.139	5.04	0.70
Asset-Backed Securities	0.017	3.16	0.05
Credit	0.274	6.35	1.79
Total	1.000		4.60

EXHIBIT 4-10	Calculation of Spread Duration and Contribution to the Lehman Brothers Aggregate Index Spread Duration: October 26, 2007		
Sector	**Weight in Index**	**Sector Spread Duration**	**Contribution to Index Spread Duration**
Treasury	0.230	0.00	0.00
Agency	0.105	3.53	0.37
Mortgages	0.381	3.62	1.38
Commercial Mortgage-Backed Securities	0.056	5.04	0.28
Asset-Backed Securities	0.010	3.16	0.03
Credit	0.219	6.53	1.43
Total	1.000		3.49

Exhibit 4-11
Measures of Bond Price Volatility and Their Relationships to One Another

Notation:

D = *Macaulay duration*

D^* = *modified duration*

$PVBP$ = price value of a basis point

y = *yield to maturity in decimal form*

Y = *yield to maturity in percentage terms* ($Y = 100 \times y$)

P = *price of bond*

m = *number of coupons per year*

Exhibit 4-11
Measures of Bond Price Volatility and Their Relationships to One Another (continued)

Relationships:

$D^* = \dfrac{D}{1 + y/m}$ → by definition

$\dfrac{\Delta P/P}{\Delta y} \approx D^*$ → to a close approximation for a small Δy

$\Delta P/\Delta Y \approx$ slope of price-yield curve → to a close approximation for a small Δy

$PVBP \approx \dfrac{D^* \times P}{10{,}000}$ → to a close approximation

For Bonds at or near par:

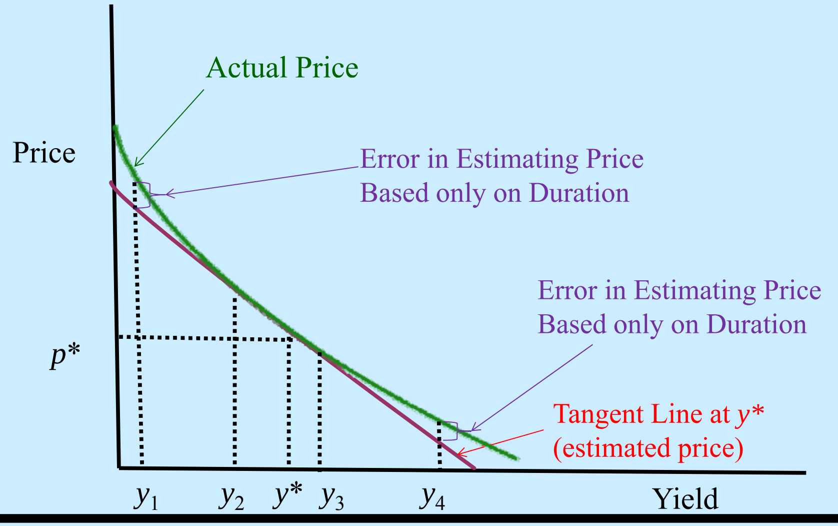
$PVBP = D^*/100$ → to a close approximation

$D^* = \Delta P/\Delta Y$ → to a close approximation for a small Δy

Why Need Convexity?

- All the duration measures are only approximations for small changes in yield,
  - Do not capture the effect of the convexity
- The duration measure can be supplemented with an additional measure to capture the curvature (or convexity)
- In Exhibit 4-13 (*next slide*), a tangent line is drawn to the price–yield relationship at yield y^*.
- The approximation will always understate the actual price.
  - When yields decrease, the estimated price change will be less than the actual price change, thereby underestimating the actual price.
  - When yields increase, the estimated price change will be greater than the actual price change, resulting in an underestimate of the actual price

Exhibit 4-13. Price Approximation Using Duration



Measuring convexity

- Use the first two terms of a Taylor series to approximate the price change of a bond

$$dP = \frac{dP}{dy}dy + \frac{1}{2}\frac{d^2P}{dy^2}(dy)^2 + error$$

Dollar convexity measure

The dollar price change based on dollar duration

- The percentage price change

$$\frac{dP}{P} = \frac{dP}{dy}\frac{1}{P}dy + \frac{1}{2}\frac{d^2P}{dy^2}\frac{1}{P}(dy)^2 + \frac{error}{P}$$

Convexity measure

- Measures of convexity

$$dollar\ convexity\ measure = \frac{d^2P}{dy^2}$$

$$convexity\ measure = \frac{d^2P}{dy^2}\frac{1}{P}$$

Calculating Convexity

- The percentage price change due to convexity

$$\frac{dP}{P} = \frac{1}{2}(convexity\ measure)(dy)^2$$

- The second derivative of the bond price equation

$$\frac{d^2P}{dy^2} = \sum_{t=1}^{n}\frac{t(t+1)C}{(1+y)^{t+2}} + \frac{n(n+1)M}{(1+y)^{n+2}}$$

- Converting the convexity measures to an annual figure
  - Convexity is measured in terms of periods

$$convexity\ in\ years = \frac{convexity\ in\ m\ periods\ per\ year}{m^2}$$

Calculating Convexity

EXHIBIT 4-10 Calculation of Convexity Measure and Dollar Convexity Measure for Five-Year 9% Bond Selling to Yield 9%

Coupon rate: 9.00%
Term (years): 5
Initial yield: 9.00%
Price: 100

Period, t	Cash Flow[a]	$\frac{1}{(1.045)^{t+2}}$	$t(t+1)CF$	$\frac{t(t+1)CF}{(1.045)^{t+2}}$
1	$4.50	0.876296	9	7.886
2	$4.50	0.838561	27	22.641
3	$4.50	0.802451	54	43.332
4	$4.50	0.767895	90	69.110
5	$4.50	0.734828	135	99.201
6	$4.50	0.703185	189	132.901
7	$4.50	0.672904	252	169.571
8	$4.50	0.643927	324	208.632
9	$4.50	0.616198	405	249.560
10	$104.50	0.589663	11,495	6,778.186
			12,980	7,781.020

[a] Cash flow per $100 of par value.

Second derivative = 7, 781.02

$$\text{Convexity measure (half-years)} = \frac{7,781.020}{100.0000} = 77.8102$$

$$\text{Convexity measure (years)} = \frac{77.8102}{4} = 19.4526$$

Dollar convexity measure = 100 × 19.4526 = 1,945.26

Calculating percentage price changes using both duration and convexity

- Approximate percentage price change for a given change in the yield

$$\frac{dP}{P} = -(\text{modified duration})(dy) + \frac{1}{2}(\text{convexity measure})(dy)^2$$

  - price change = duration estimate + convexity adjustment
  - The convexity adjustment gets the estimate closer to the actual price

Calculating percentage price changes using both duration and convexity

- Example
  - A 25-year 6% bond selling at 70.3570 to yield 9%. The modified duration is 10.62 and the convexity measure is 182.92. Assuming yield increase by 200 basis points, from 9% to 11%.
  - % change in price applying duration and convexity

    = -(modified duration) (dy)+1/2(convexity measure) $(dy)^2$

    = $-10.62\times0.02 + 0.5\times182.92\times0.02^2$

    = -21.24% + 3.66%

    = -17.58%
  - The actual change is -18.03%

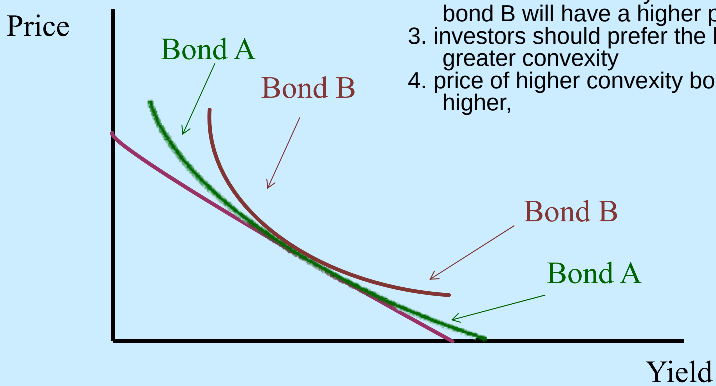
Value of Convexity

The convexity of a bond has another important investment implication illustrated in Exhibit 4-16 *(see next slide)*.

- The two bonds, A and B, have the same duration and are offering the same yield
- Bond B is more convex than bond A.
- The market price reflects bond's convexity.
- If investors expect that market yields will change very little, he will not pay much for convexity.
- If the market prices convexity high, investors with expectations of low interest rate volatility will probably want to "sell convexity."

Exhibit 4-16
Comparison of Convexity of Two Bonds

1. Bond B has greater convexity
2. Whether the market yield rises or falls, bond B will have a higher price
3. investors should prefer the bond with greater convexity
4. price of higher convexity bond should be higher,

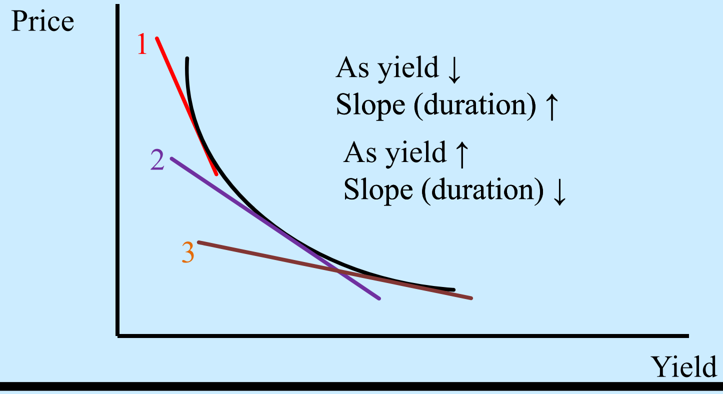


Properties of Convexity

- All option-free bonds have the following convexity properties illustrated in Exhibit 4-17 *(see next slide),*

i. the required yield increases (decreases), the convexity of a bond decreases (increases). This property is referred to as positive convexity.
ii. For a given yield and maturity, lower coupon rates will have greater convexity. (illustrated 2 pages later)
    i. zero coupon bond has the highest convexity
iii. For a given yield and modified duration, lower coupon rates will have smaller convexity.

Exhibit 4-17
Change in Duration as the Required Yield Changes



Properties of Convexity

For same yield and maturity, zero bonds are more convexity		For same duration and yield, zero-Coupon bonds are less convexity	
A	B	A	B
N=6	N=6	N=6	N=5
Y=8%	Y=8%	Y=8%	Y=8%
C=8%	C=0%	C=8%	C=0%
D=5	D=6	D=5	D=5
CX=28	CX=36	CX=28	CX=25.72

A and B are two different bond. C is coupon rate, Y is yield of maturity, D is duration, and CX is convexity.

Additional Concerns when Using Duration

- Relying on duration as the sole measure of the bond price volatility could be misleading.
- Two concerns should be point out:

i. First, we assume that all cash flows for the bond are discounted at the same discount rate.
    - Flat yield curve assumption
    - Parallel shift of yield curve
ii. Second, there is misapplication of duration to bonds with embedded options.

Don't Think of Duration as a Measure of Time

- ❖ Market participants often confuse the main purpose of duration by constantly referring to it as "the weighted average life of a bond".
- ❖ CMO (collateralized mortgage obligation) are leveraged instruments:
  - – Price sensitivity or duration are a multiple of the underlying mortgage loans
- ➢ A CMO bond class with a duration of 40 does not mean that it has some type of weighted average life of 40 years.
- ➢ It means that for a 1% change in yield, that bond's price will change by roughly 40%.
- ❖ Like a CMO bond class, we interpret the duration of an option in the same way.

Numerically Approximating a Bond's Duration and Convexity

- ❖ A simple formula to calculate the approximate duration of a bond or any other more complex derivative securities or options.
- ❖ The formula shows the percentage price change of a bond when interest rates change by a small amount:

$$\text{approximate duration} = \frac{P_{-} - P_{+}}{2(P_0)(\Delta y)}$$

where Δy is the change in yield used to calculate the new prices.

- ➢ The above formula measures the average percentage price change (relative to the initial price) per 1-basis-point change in yield.
- ❖ The convexity measure of any bond can be approximated using the following formula:

$$\text{approximate convexity measure} = \frac{P_{+} + P_{-} - 2P_0}{P_0(\Delta y)^2}$$

Ex: Evaluating the Duration

- – Increase the yield on the bond by a small number of basis point (10 bps) from 9% to 9.1%, recalculate bond price using bond pricing formula, the new price P_+ is 69.6164.
- – Decrease the yield on the bond by a small number of basis point (10 bps) from 9% to 8.9%, recalculate bond price using bond pricing formula, the new price P_- is 71.1105.
- – Because the initial price, P_0, is 70.3570, the duration can be approximated as follows:

$$\textit{approximate duration} = \frac{P_{-} - P_{+}}{2P_0\Delta y} = \frac{71.1105 - 69.6164}{2\times 70.3570\times 0.001} = 10.62$$

Ex: Evaluating the Convexity

- Convexity measure can be calculated

$$\textit{approximate convexity measure} = \frac{P_{+} + P_{-} - 2P_0}{P_0(\Delta y)^2}$$

$$= \frac{71.1105 + 69.6164 - 2\times(70.3570)}{70.3570\times(0.001)^2} = 183.3$$

Duration of floaters and inverse floaters

- A floater and an Inverse floater can be created from their underlying collateral bond
- Duration of the underlying collateral bond D_c is the weighted average of D_f and D_i

$$w \times D_i + (1 - w) \times D_f = D_c$$

- Duration of a floater is close to 0
  - Price of floater is close to its par value, regardless the change of yield

$$w \times D_i = D_c$$

$$D_i = \frac{D_c}{w} = D_c \times \frac{\text{Value(Collateral)}}{\text{Value(Inverse)}} = D_c \times \frac{\text{Par(Collateral)} \times \text{Collateral Price}}{\text{Par(Inverse)} \times \text{Inverse Price}}$$

$$= \left(1 + \frac{\text{Par(}F\text{loater)}}{\text{Par(Inverse)}}\right) \times D_c \times \frac{\text{Collateral Price}}{\text{Inverse Price}}$$

Measuring a Bond Portfolio's Responsiveness to Nonparallel Changes in Interest Rates

❖ **Yield Curve Reshaping Duration**

➢ The yield curve reshaping duration approach focuses on the sensitivity of a portfolio to a change in the slope of the yield curve.

❖ **Key Rate Duration**

➢ The most popular measure for estimating the sensitivity of a portfolio to the changes in the yield curve

➢ Change the yield for a particular maturity and determine the sensitivity of a portfolio given all other yields constant.