## A Closer Look at Production and Costs

 CHAPTER7 Appendix

## The Production Function and Efficiency

* The production function identifies the maximum quantities of a particular good or service that can be produced per time period with various combinations of resources, for a given level of technology


## Production Function Using Labor and Capital

| Units of Capital | Units of Labor Employed per Month |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Employed per Month | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ |
| $\mathbf{1}$ | $\mathbf{4 0}$ | $\mathbf{9 0}$ | $\mathbf{1 5 0}$ | $\mathbf{2 0 0}$ | $\mathbf{2 4 0}$ | $\mathbf{2 7 0}$ | $\mathbf{2 9 0}$ |
| $\mathbf{2}$ | $\mathbf{9 0}$ | $\mathbf{1 4 0}$ | $\mathbf{2 0 0}$ | $\mathbf{2 5 0}$ | $\mathbf{2 9 0}$ | $\mathbf{3 1 5}$ | $\mathbf{3 3 5}$ |
| $\mathbf{3}$ | $\mathbf{1 5 0}$ | $\mathbf{1 9 5}$ | $\mathbf{2 6 0}$ | $\mathbf{3 1 0}$ | $\mathbf{3 4 5}$ | $\mathbf{3 7 0}$ | $\mathbf{3 9 0}$ |
| $\mathbf{4}$ | $\mathbf{2 0 0}$ | $\mathbf{2 5 0}$ | $\mathbf{3 1 0}$ | $\mathbf{3 5 0}$ | $\mathbf{3 8 5}$ | $\mathbf{4 1 5}$ | $\mathbf{4 4 0}$ |
| $\mathbf{5}$ | $\mathbf{2 4 0}$ | $\mathbf{2 9 0}$ | $\mathbf{3 4 5}$ | $\mathbf{3 8 5}$ | $\mathbf{4 2 0}$ | $\mathbf{4 5 0}$ | $\mathbf{4 7 5}$ |
| $\mathbf{6}$ | $\mathbf{2 7 0}$ | $\mathbf{3 2 0}$ | $\mathbf{3 7 5}$ | $\mathbf{4 1 5}$ | $\mathbf{4 5 0}$ | $\mathbf{4 7 5}$ | $\mathbf{4 9 5}$ |
| $\mathbf{7}$ | $\mathbf{2 9 0}$ | $\mathbf{3 3 0}$ | $\mathbf{3 9 0}$ | $\mathbf{4 3 5}$ | $\mathbf{4 7 0}$ | $\mathbf{4 9 5}$ | $\mathbf{5 1 0}$ |

* The firm produces the maximum possible output given the combination of resources employed $\rightarrow$ production is technologically efficient
* Compute marginal product of labor (given capital=1)
$1 \rightarrow 40$
$2 \rightarrow 90-40=50$
a $3 \rightarrow 150-90=60$
$5 \rightarrow 240-200=40$
$4 \rightarrow 200-150=50$
$6 \rightarrow 270-240=30$
$8 \rightarrow 290-270=20$
- 


## Production Function

| Units of Capital | Units of Labor Employed per Month |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Employed per Month | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ |
| $\mathbf{1}$ | $\mathbf{4 0}$ | $\mathbf{9 0}$ | $\mathbf{1 5 0}$ | $\mathbf{2 0 0}$ | $\mathbf{2 4 0}$ | $\mathbf{2 7 0}$ | $\mathbf{2 9 0}$ |
| $\mathbf{2}$ | $\mathbf{9 0}$ | $\mathbf{1 4 0}$ | $\mathbf{2 0 0}$ | $\mathbf{2 5 0}$ | $\mathbf{2 9 0}$ | $\mathbf{3 1 5}$ | $\mathbf{3 3 5}$ |
| $\mathbf{3}$ | $\mathbf{1 5 0}$ | 195 | $\mathbf{2 6 0}$ | $\mathbf{3 1 0}$ | $\mathbf{3 4 5}$ | $\mathbf{3 7 0}$ | $\mathbf{3 9 0}$ |
| $\mathbf{4}$ | $\mathbf{2 0 0}$ | $\mathbf{2 5 0}$ | $\mathbf{3 1 0}$ | $\mathbf{3 5 0}$ | $\mathbf{3 8 5}$ | $\mathbf{4 1 5}$ | $\mathbf{4 4 0}$ |
| $\mathbf{5}$ | $\mathbf{2 4 0}$ | $\mathbf{2 9 0}$ | $\mathbf{3 4 5}$ | $\mathbf{3 8 5}$ | $\mathbf{4 2 0}$ | $\mathbf{4 5 0}$ | $\mathbf{4 7 5}$ |
| $\mathbf{6}$ | $\mathbf{2 7 0}$ | $\mathbf{3 2 0}$ | $\mathbf{3 7 5}$ | $\mathbf{4 1 5}$ | $\mathbf{4 5 0}$ | $\mathbf{4 7 5}$ | $\mathbf{4 9 5}$ |
| $\mathbf{7}$ | $\mathbf{2 9 0}$ | $\mathbf{3 3 0}$ | $\mathbf{3 9 0}$ | $\mathbf{4 3 5}$ | $\mathbf{4 7 0}$ | $\mathbf{4 9 5}$ | $\mathbf{5 1 0}$ |

- Marginal product of labor first rise $\rightarrow$ increasing marginal returns
* Then declines $\rightarrow$ diminishing marginal returns
- So does the marginal product of capital.
- Different combinations of resources yield the same rate of output
${ }^{3}$ : Several combinations of labor and capital yield 290 units of output
m Next slide provides another perspective on this information



## Properties of Isoquants

## * A unique isoquant for every output rate

## Along a given isoquant, the quantity of labor employed is inversely related to the quantity of capital employed 진 isoquants have negative slopes

## Properties of Isoquants

* I soquants do not intersect.
s an intersection would indicate that the same combination of resources produce two different amounts of output $\rightarrow$ I nefficient
s I soquants are usually convex to the origin


## Marginal Rate of Technical Substitution (MRTS)

* The absolute value of the slope of the isoquant
娄 = marginal rate of technical substitution, MRTS,
- Thus, the MRTS is the rate at which labor substitutes for capital without affecting output
See next slide


## When Isoquant is Convex

| When much capital and little <br> labor are used, |
| :--- |
| It takes much capital to make up <br> for a one-unit reduction in labor |
| -The marginal productivity of <br> labor is relatively large <br> -The marginal productivity of <br> capital relatively small |
| As more labor and less capital <br> are used, <br> -The marginal product of <br> labor declines <br> -The marginal product of <br> capital increases |

## MRTS and Marginal Productivity

## MRTS is directly linked to the marginal productivity of each input

From $a \rightarrow b$,
1 unit of labor replaces 2 units of capital,
n labor's marginal product, $M P_{L}$ must be twice as large as capital's marginal product, $M_{C}$
We conclude MRTS $=$ MP $_{\mathrm{L}} / \mathrm{MP}_{\mathrm{C}}$
e If labor and capital were perfect substitutes in production, 연 MRTS will be fixed constant
\% The isoquant would be a downward sloping straight line

## Minimize Production Cost

 Construct Isocost Lines* I ntuition:
a Given rate of output
- Combination of resources along Isoquant
a Find one combination that minimize the production cost
© Suppose
a 4 anit of labor $=\$ 1,500$
a unit of capital costs \$2,500
s TC $=\$ 1,500 \mathrm{~L}+\mathbf{2 , 5 0 0 C}$
* See next slide

| Construct Isocost Lines |
| :--- | :--- | :--- |
| Isocost line identifies |
| all combinations of |
| capital and labor for a |
| given total cost |

## Choice of Input Combinations

* To maximizes profit, firm produces its chosen output at the minimum cost
* Next figure brings together
as isoquants line $\rightarrow$ Chosen output
$a$ isocost line $\rightarrow$ Minimize cost


## Optimal Combinations of Inputs

Cost minimize at point $e$,
The isoquant is just tangent to the isocost line of $\mathbf{\$ 1 9 , 0 0 0}$

At $e$, the isoquant and isocost line have the same slope MRTS= the ratio of resources prices

MRTS $=\mathrm{w} / \mathrm{r}=\mathbf{1 , 5 0 0} / \mathbf{2 , 5 0 0}=$ 0.6


## Expansion Path

## Given

a a set of isoquants
a relative cost of resources,

* we can determine the optimal
combination of resources for producing each rate of output
[al The points of tangency between
- isoquant line
- isocost line.
show the least-cost input combinations for producing several output rates
These points form an expansion path.
See next slide


## Graph of Expansion Path

Expansion path $\rightarrow$ the lowest longrun total cost for each rate of output.

Long-run average cost curve indicates, at each rate of output, the total cost divided by the rate of output

The expansion path and the longrun average cost curve are alternative ways of portraying costs in the long run



## Short Run and the Long Run Adjustment

The firm decide to produce $Q_{2}$ at point $b$ requires
$C$ units of capital
$L$ units of labor.
In the short run, the firm wants to increase output to $\mathbf{Q}_{3}$.

Capital is fixed (C) in the short run, Increasing labor to $\mathrm{L}^{\text {‘ }}$ (point $e$.)

Point $e$ is not the cheapest way to produce $Q_{3}$ because it is not a tangency point.

In the long run, capital is variable,
It should minimize total cost by adjusting from point $e$ to point $c$



## Expansion Path

© Relative prices of resources change,
$\rightarrow$ the least-cost resource combination will also change
$\rightarrow$ the firm's expansion path will change

- If the price of labor increases, $\rightarrow$ capital becomes relatively less expensive
$\rightarrow$ the efficient production will call for less labor and more capital

