Indifference Curves and Utility Maximization

CHAPTER

6 Appendix

1

A New Approach for Utility Maximization

- Marginal utility analysis requires numerical measure of utility to determine the optimal consumption combinations

- A new approach is developed to analyze utility and consumer behavior
  - It does not require that numbers be attached to specific levels of utility

- Assume the marginal utility of consuming each good >0 from now on.

2

Consumer Preferences

- This new approach requires
  - consumers be able to rank their preferences for various combinations of goods

- Consumer should be able to say whether
  - Combination A is preferred to combination B
  - Combination B is preferred to combination A. or
  - Both combinations are equally preferred

3

Indifference Curves and Utility Maximization

- *Indifference curve* shows all combinations of goods that provide the consumer with the same satisfaction, or the same utility

- Thus, the consumer finds all combinations on a curve equally preferred

- Since each combinations of goods yields the same level of utility,
  - →the consumer is *indifferent* about which combination is actually consumed

4

An Indifference Curve

•**Only two goods available: pizzas and movie videos**

•**Point *a* shows the consumption bundle consisting of 1 pizza and 8 video rentals**

•**Holding utility constant, how many video rentals would a person be willing to give up to get a second pizza?**

•**Moving from point *a* to point *b*, we are willing to give up 4 videos to get a second pizza (total utility is the same at points *a* and *b*);**
•**the marginal utility of another pizza is just sufficient to compensate for the utility lost from decreasing video purchases by 4 movies.**
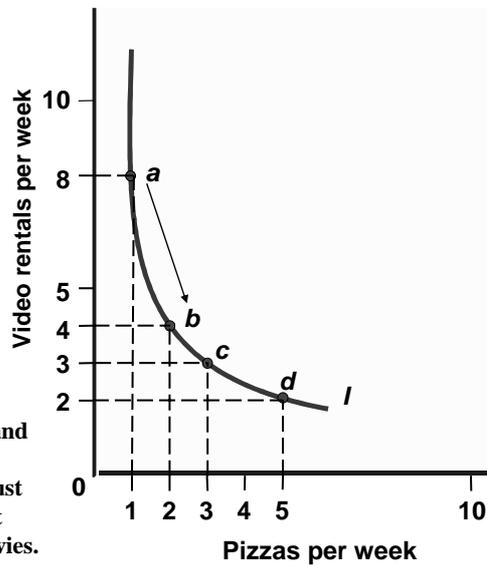

5

An Indifference Curve

From *b* → *c*, again total utility is constant; the person is willing to give up only 1 video for another pizza.
From *c* → *d*, the person is willing to give up another video only if they get two more pizzas in return

Points *a*, *b*, *c*, and *d* can be connected to form the *indifference curve, I, which represents possible combinations of pizza and videos that would keep the person at the same level of total utility*.

Combinations of goods along an indifference curve reflect a constant level of total utility

6

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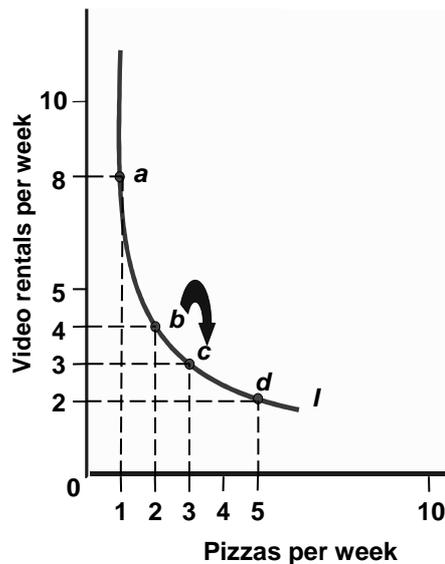
Indifference Curves

- **On a indifferent curve, the increase in utility from eating more pizza must just offset the decrease in utility from watching fewer videos**

- **Thus, along an indifference curve, there is an inverse relationship between the quantity of one good consumed and the quantity of another consumed ➔ *indifference curves slope down***

Features of Indifference Curves

- **Indifference curves are also convex to the origin ➔ they are bowed inward toward the origin**

- **The curve gets flatter as you move down it**

- ***The marginal rate of substitution, or MRS***
  - **maintaining the same level of total utility**
  - **the number of videos that the consumer is willing to give up to get one more pizza,**

Marginal Rate of Substitution

- **The MRS measures the consumers willingness to trade videos for pizza**
  - It depends on the amount of each good the consumer is consuming at the time

- **MRS is equal to the absolute value of the slope of the indifference curve**
  - From a➔ b, the consumer is willing to give up 4 videos to get 1 more pizza
  - ➔ slope = −4
  - ➔ MRS = 4
  - From b➔ c, MRS=1

Law of Diminishing Marginal Rate of Substitution

- **As a persons consumption of pizza increases, the number of videos that they are willing to give up to get another pizza declines**

- **This implies MRS decreases**
  - Move down the indifference curve,
    - the marginal utility of additional pizza declines
    - the marginal utility of additional video increases

Indifference Map

 Generate a series of indifference curves, called an indifference map ➔ graphical representation of a consumer's tastes

 Each curve in the map reflects a different level of utility

 See next slide

An Indifference Map

Each indifference curve represents a different level of utility
Each consumer has a unique indifference map based on their preferences

Curves farther from the origin represent higher levels of utility
➔ Total utility along I_2 higher than along I_1, I_3 higher than I_2, etc

This can be verified by drawing a ray from the origin and following it to higher indifference curves

Video rentals per week

10 —

5 —

I_4

I_3

I_2

I_1

0   5   10

Pizzas per week
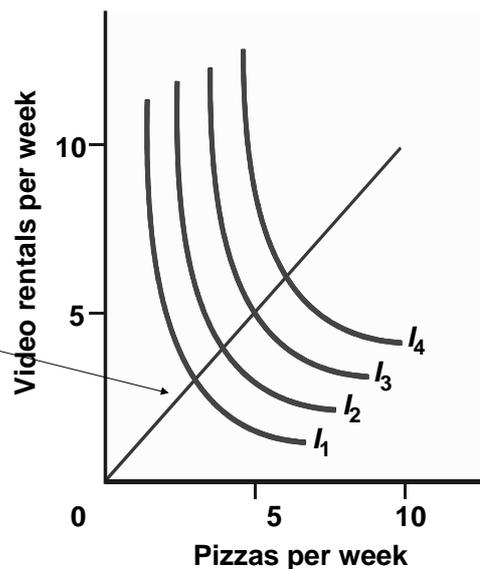
Indifference Curves Do Not Intersect

If indifference curves crossed, such as point *i*, i, j, k will have the same utility.

But point *k* is a combination with more pizza and more videos than point *j* ➔ must represent a higher level of utility
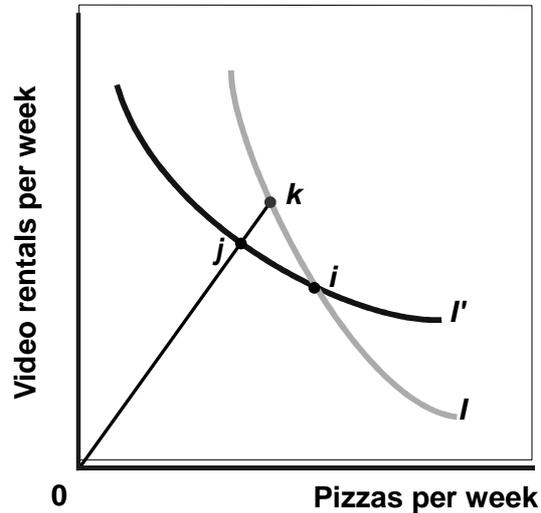
13

How we find the equilibrium point of consumption

- **Once we have the consumer's indifference may, we turn to the issue of how much of each good will be consumed?**

- **To answer this question, we must consider the relative prices of the two goods and the consumer's income**

14

Budget Line

- **Budget line depicts all possible combinations of movies and pizzas, given prices and your budget**

- **Suppose movies rent for $4, pizza sells for $8, and the budget is $40 per week**
  - **if you spend the entire $40 on videos, consumer can purchase 10 videos,**
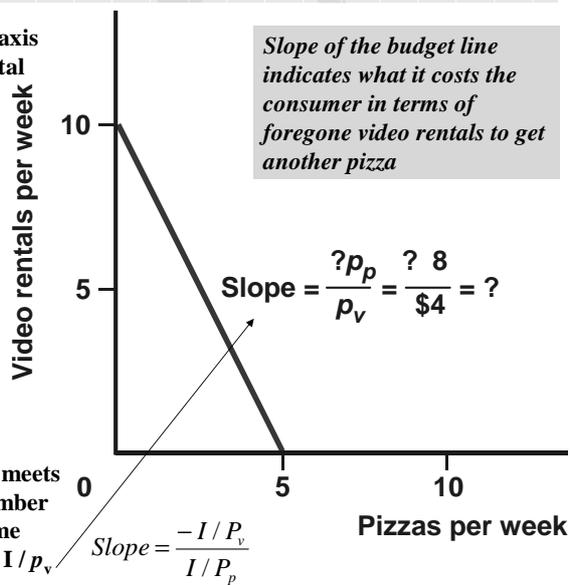 - **if you spend the entire $40 on pizzas person can afford 5 per week**

15

A Budget Line

The budget line meets the vertical axis at 10 videos and meets the horizontal axis at 5 pizzas

These two intercepts are then connected to form the budget line.

The budget line defines all possible combinations of pizza and videos, that can be purchased,
➜ can be thought of as a *consumption possibilities frontier*

At the point where the budget line meets the vertical axis, the maximum number of videos you can rent equals income divided by the video rental price = I / p_v and for the horizontal axis is I / p_p

Slope of the budget line indicates what it costs the consumer in terms of foregone video rentals to get another pizza

Video rentals per week

10

5

$$\text{Slope} = \frac{?p_p}{p_v} = \frac{?\,8}{\$4} = ?$$

0   5   10

Pizzas per week

$$Slope = \frac{-I / P_v}{I / P_p}$$

16

8

Summary

- The indifference curve indicates what the consumer is *willing* to buy

- The budget line shows what the consumer is *able* to buy

- When the indifference curve and the budget line are combined, we find the quantities of each good the consumer is both *willing and able* to buy

- See next slide

17

Utility Maximization

The utility-maximizing consumer will select a combination along the budget line that lies on the highest attainable indifference curve

Given prices and income, this occurs at point *e*, where I_2 just touches, or is tangent to, the budget line

Other attainable combinations along the budget line reflect lower levels of utility

18

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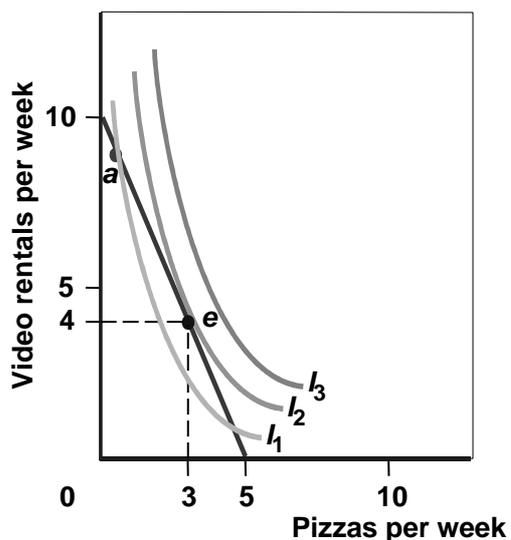
Consumer Equilibrium

- Consumer equilibrium occurs:

  slope of the indifference curve =
   slope of the budget line

- The absolute value of the slope of the indifference curve
  - = the marginal rate of substitution,
- The absolute value of the slope of the budget line
  - = the price ratio

19

Consumer Equilibrium

- MRS = Slope of budget line P_p / P_v

- The marginal rate of substitution of pizzas for video rentals
  - the number (n) of videos that the consumer is willing to give up to get one more pizza
  - $MU_p = n \times MU_v$
- Use marginal utilities of pizza and video ➜ MRS = MU_p / MU_v

20

Consumer Equilibrium

- slope of the indifference curve =
  slope of the budget line
  $MU_p/MU_v = p_p / p_v$

$$\frac{MU_P}{P_P} = \frac{MU_V}{P_V}$$

Effects of a Change in Price

- What happens to the consumer's equilibrium consumption when there is a change in price?

- How to use indifference curve approach to derive the demand curve?
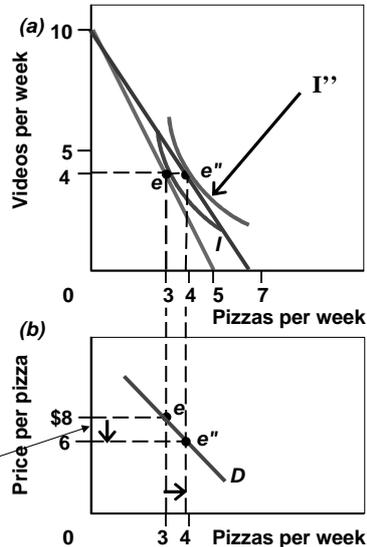
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Effect of a Drop in Price of Pizza

The demand curve is shown in the lower panel and depicts how price and quantity demanded are related

Price of pizza $8 , purchase 5 (40/8) pizzas.
Price of pizza $6,   the consumer could purchase

6.67 pizzas (40 / 6) pizzas.
Equilibrium : e→ e''
the quantity demanded increases from 3 to 4

Since the consumer is on a higher indifference curve at *I''*, the consumer is clearly better off
➔ The consumer surplus has increased

(a)

Videos per week

10

5
4

I''

e''
e

I

0   3 4 5   7   Pizzas per week

(b)

Price per pizza

$8
6

e
e''

D

0   3 4   Pizzas per week

23

Income and Substitution Effects

- **The law of demand was initially explained in terms of an income effect and a substitution effect**

- **With indifference curve analysis we have the analytical tools to examine these two effects more precisely**

- **See next slide**
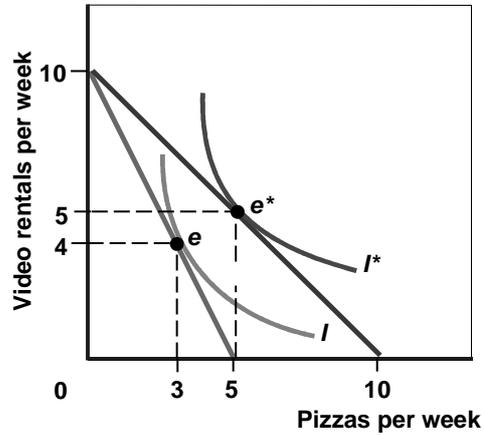
24

Substitution and Income Effects
*Equilibrium e → e**

Suppose the price of pizza falls from $8 to $4, other things constant → consumer can now purchase a maximum of 10 pizzas with a budget of $40.

e→e*
Quantity demand: 3→5

The increase in the quantity of pizzas demanded can be broken down into the substitution and the income effect of a price change.

To derive the substitution effect, let's assume that you must maintain the same level of utility after the price change as before → consumer's utility level has not changed but the relative prices you face have changed.

25

Substitution and Income Effects
Substitution Effect

A new budget line reflecting the change in relative prices, not a change in utility is shown by the dashed line CF.

Utility is at the initial level with the new relative prices, but adjust income so that utility remains the same

The consumer moves down along the indifference curve I to point *e'*, renting fewer videos but buying more pizzas. These changes reflect the substitution effect of lower prices of pizza
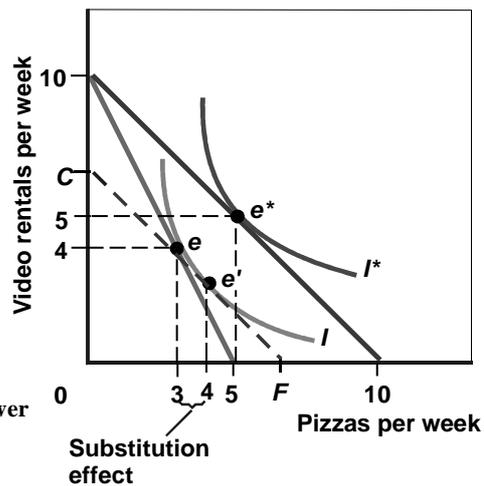
26

Substitution and Income Effects

Income Effects

But at point *e'*, the consumer has not spent the full budget. The consumer's real income has increased because of the lower price of pizza
➔ he is able to attain point *e** on indifference curve I*.

Because relative prices are held constant during the move from *e'* to *e**, the change in consumption is due solely to a change in real income ➔ the change in the quantity demanded from 4 to 5 reflects the income effect of the lower pizza price

27