Futures and Forwards

Financial Engineering and Computations

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Outline

- Brief Introduction of Derivatives
- Forwards
- Relationships Between Forward Price and Spot Price
- Futures
- Relationships between Forward and Futures Prices
- Cost of Carry
- Hedging and Futures
- Minimum Variance Hedge Ratio

Derivatives

- Payoffs depend on other more fundamental assets:
  - Underlying assets
    - Commodity
    - Index
    - Interest rate
    - Other derivatives
- Four types of derivatives
  - Forwards(遠期契約)
  - Futures(期貨)
  - Swaps(交換合約)
  - Options(選擇權)

遠期契約 (Forwards)

- 定義：遠期契約雙方同意在未來某一天以某一價格(履約價格)交易某一特定資產或商品。

- Terms:
  Maturity date:到期日
  Strike price:履約價格
  Underlying asset: 標的物

Why we Need Forwards?
A Simple Story

- 考慮XYZ公司欲投資x元生產棉花,生產期爲三個月
- 三個月後:
  - 棉花價格上升:公司大賺一筆
  - 棉花價格狂洩:公司虧本甚至倒閉
- 如何避免棉花的價格風險?

A Simple Story on Forwards

- 假定有另一紡布公司ABC,三個月後需要棉花紡布
  - 棉花不能今日進貨:倉儲問題
  - 三個月後進貨
    - 棉花價格上升:公司有虧本甚至破產之虞
    - 棉花價格狂洩:公司大賺一筆
  - 如何規避棉花的價格風險?

A Simple Story on Forwards

- XYZ和ABC之間可簽訂合約
  - 使用預定好的金額K於三個月後交易棉花
  - 價格風險消失,但也無法得額外利潤
- XYZ (Forward的報酬爲 K-P)

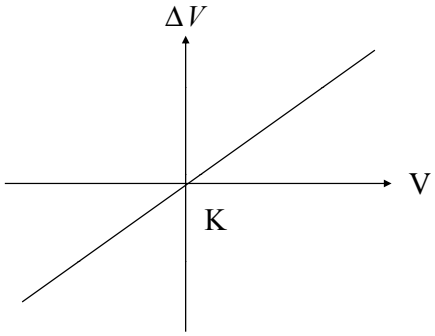
	不簽合約	簽合約	
		遠期合約	市場價格
價格上升至P1	P1(額外利潤)	K-P1	P1
價格下降至P2	P2(虧損)	K-P2	P2

總報酬鎖定爲K

A Simple Story on Forwards

- 上述爲一遠期合約
- 標的物:棉花
- 到期日:三個月後(假定棉花價格爲P)
- 履約價格:K
- 買方:ABC (報酬爲 P-K)
- 賣方:XYZ (報酬爲 K-P)
- 交易方式:
  - 實物交易
  - 現金清算

從 Buyer (ABC)來看 Forward的報酬



1. 報酬構面(payoff profile)是直線的
2. 遠期合約期初價值爲零，期中無任何支付，僅有期末會有支付產生。

Forward的缺點

- 當棉花價格上升,XYZ失去大賺一筆的機會
- 當棉花價格下降,ABC失去減少成本的機會
  - Options
- Default risk(違約風險)
  - XYZ可能失火,導致無法履約
  - 當棉花價格上升,XYZ有違約的可能
  - 當棉花價格下降,ABC有違約的可能
    - 採用期貨合約每日清算的制度,減少違約風險

Forward的缺點

- 合約未標準化
  - 採用標準化的期貨合約
  - 以台指期貨爲例
    - 交易標的物:加權指數
    - 規格:近三個月和兩個季月到期的合約
      - Ex: Now: Oct., (Introduced later)
      - 交易合約到期日: Oct., Nov., Dec., Mar., Jun.
- 市場撮合和流通
  - 期貨合約可在交易所流通

Reasonable Strike Price

- 在理想的假定下(no tax, transaction cost, etc.)遠期的履約價格和標的物的現貨價格有一定的關係
- 當該關係不滿足時,存在套利機會
- 凱因斯(Keynes)使用 Interest parity指出該狀況
  - 考慮遠期匯率合約
  - 匯率和本國利率,外國利率有連動的關係

Interest Rate Parity

- Terms:
  - S: Spot exchange rate (Domestic/Foreign)
  - F: Forward exchange rate (Maturity: T year)
  - R_f: Foreign interest rate
  - R_l: Local interest rate
- Then we have $\frac{F}{S} = e^{(R_l - R_f)T}$
- Otherwise, there exists arbitrage opportunity.

Arbitrage Opportunity

- Assume $\frac{F}{S} > e^{(R_l - R_f)T}$
- Now:
  - Borrow 1 domestic dollar, change to 1/S foreign dollar, save it at a interest rate of Rf
  - Sign a exchange rate forward with price F.
- T year later:
  - You get $\frac{1}{S}e^{R_f T}$ foreign dollars.
  - Convert them at the rate F (Forward contract)
  - Since $\frac{F}{S}e^{R_f T} > e^{R_l T}$ → get free lunch

In Class Exercise

- Show that arbitrage opportunity exists as $\frac{F}{S} < e^{(R_l - R_f)T}$

Forward Price

- 遠期合約在期初時不會有現金流
  - 遠期合約初始價值爲0 (買方不需付權利金給賣方)
- Forward price (F) 代表一合理的價格,使得當履約價格(K)等於F時 ,遠期合約的初始價值爲0
- Forward price會隨時間而改變
  - 令$\mathbf{F_t}$爲時間t的Forward price
  - 假定合約起始日爲0,到期日爲T
  - $\mathbf{S_T}$爲標的物在時間T的價格
  - 可類推到期貨(future price)

Contract Value

- 遠期合約在期初的價值爲0,
- 遠期合約在時間t的價值爲 $f_t = S_t - Ke^{-r(T-t)}$
- Proof:
  - At time t
    - Long a forward contract
    - Save $Ke^{-r(T-t)}$
    - Short underlying asset
  - At time T

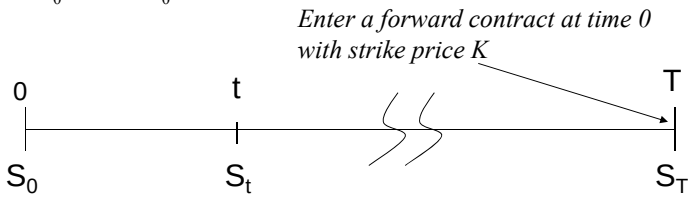
$$(S_T - K) + K - S_T = 0$$ Riskless, Benefit=0

Forward Price和Spot Price關係 (標的物於T前無支付現金)

- 在時間 0時,現貨價格爲S_0
- 假定標的物在T前不會產生 cash flow (like dividend)
- 則可得 $F_0 = S_0 e^{rT}$
- 如上式不滿足,則存在套利空間

令 $f_0 = S_0 - Ke^{-r(T-0)} \equiv 0$

$F_0 = K = S_0 e^{rT}$



Arbitrage Opportunity

- 假定 $F_0 > S_0 e^{rT}$
- 時間0時簽合約,K= F_0

Cash flow

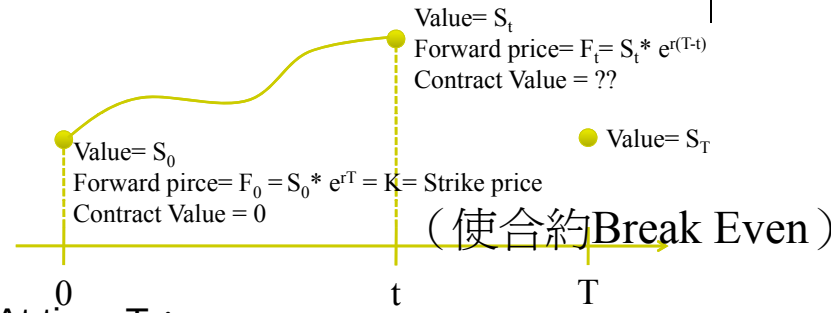
時間0	借S0	Buy the stocks	Short Forward	0
時間T	還 $S_0 e^{rT}$	得S_T	還S_T-K ($K=F_0$)	F_0-S_0e^{rT}

$-S_0 e^{rT} + S_T - (S_T - F_0) > 0$ ，期初不用支付任何成本，期末可得正的報酬!

In Class Exercise

- 考慮 $F_0 < S_0 e^{rT}$ 的套利機制

實例:在時間t鎖住Forward contract value



- At time T：

$$
\begin{array}{lll}
 & 0: & S_T - S_0 * e^{rT} \\
-) & t: & S_T - S_t * e^{r(T-t)} \\
\hline
 & & -S_0 * e^{rT} + S_t * e^{r(T-t)}
\end{array}
$$

time 0 所簽的Forward Contract 在time T 可得到的報酬Value

在time T可多拿(or 多支付)的錢(即報酬差)

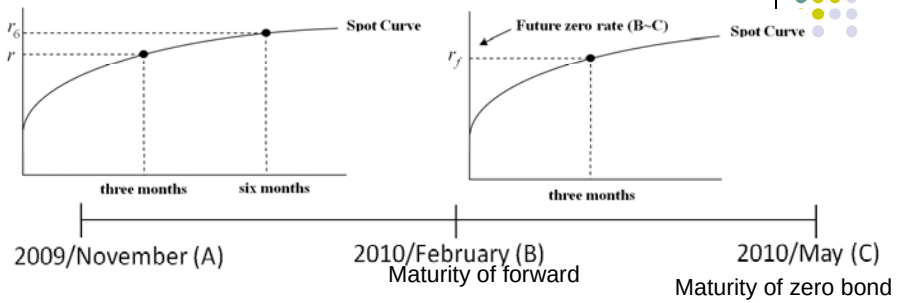
實例（續）

- 再從 time T 折回 time t的現值
- $$
\begin{aligned}
f_t &= (-S_0 * e^{rT} + S_t * e^{r(T-t)}) * e^{-r(T-t)} \\
&= S_t - S_0 * e^{rT} * e^{-r(T-t)} \\
&= S_t - K * e^{-r(T-t)}
\end{aligned}
$$
- 可在時間 t鎖住 contract value $f_t = S_t - Ke^{-r(T-t)}$

A Real Example

- r is the annualized 3-month riskless interest rate.
- S is the spot price of the 6-month zero-coupon bond.
- A new 3-month forward contract on a 6-month zero-coupon bond should command a delivery price of $Se^{r/4}$.
- So if $r = 6\%$ and $S = 970.87$, then the delivery price is $970.87 \times e^{0.06/4} = 985.54$.

Forwards on Bond and Interest Rates



F is face value of the 6-month zero-coupon bond
S_t is the spot price of the 6-month zero-coupon bond at time t

At time A:

$$S_A = F \times e^{-r_6 \frac{1}{2}} = 970.87$$

$$F_A = S_A \times e^{r \frac{1}{4}} = 970.87 \times e^{\frac{0.06}{4}} = 985.54$$

At time B:

$$S_B = F \times e^{-r_f \frac{1}{4}}$$

$$\text{reward of buyer} = S_B - 985.54$$

Forward Price和Spot Price關係 (標的物於T前支付固定報酬)

- 假定該報酬(例如股息)的現值爲 **I**
- 可得 $F_0 = (S_0 - I)e^{rT}$
- 假定 $F_0 > (S_0 - I)e^{rT}$
  - At time 0
    - short forward ($K=F_0$), long underlying asset, borrow S_0
    - Initial cash flow=0
  - At time T
    - $-S_T + F_0 + S_T + Ie^{rT} - S_0e^{rT} = F_0 - (S_0 - I)e^{rT} > 0$

Example

- Consider a 10-month forward contract on a stock with a price of \$50. We assume that the risk-free rate interest is 8% per annum for all maturity, and that dividends of \$0.75 per share are expected after three months, six months and nine months. Please calculate the forward price.

$$\because I = 0.75e^{-0.08\times 3/12} + 0.75e^{-0.08\times 6/12} + 0.75e^{-0.08\times 9/12} = 2.162$$

$$\Rightarrow F = (50 - 2.162)e^{0.08\times 10/12} = \$51.14$$

where I is the PV of the dividends

Homework

- Show that arbitrage opportunity exist if

  $F_0 < (S_0 - I)e^{rT}$

- Assume that the underlying stock pays a continuous dividend yield at rate q. Show that the forward price is $F_0 = S_0e^{(r-q)T}$

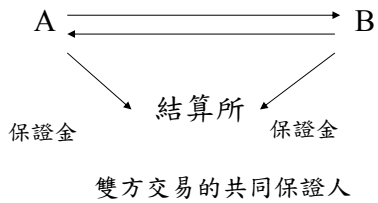
期貨合約(Futures)

- 期貨合約和遠期合約很類似，主要差別在於：

  1.每日現金結算損益(marked-to-market)

  2.買賣雙方都必須提供保證金(margin)

  —交易所藉由marked-to-market和margin account來降低違約風險



期貨合約(Futures)

3. 合約標準化(期貨均在集中交易所交易) 。
4. 透過集中市場競價來決定交易價格。
5.可以反向契約來抵銷原有契約 。

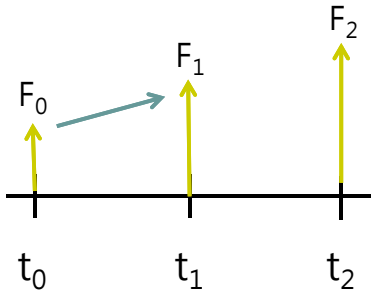
- Black: Futures可視爲一系列的遠期合約。在每日將前一天的Forward清算掉，並重新訂約。

有關期貨相關資訊可至台灣期交所 http://www.taifex.com.tw/

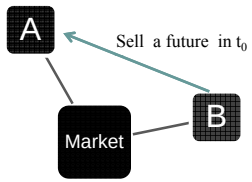
反向契約

時間軸

- 假設標的物價格隨時間上漲。
- 則每期當時的future價格也會上漲。



買賣示意圖

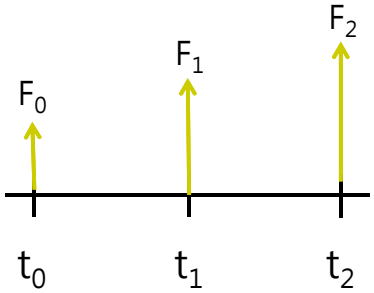


- B 在時間點 t_0 賣了一口到期日爲 t_2 的 future 給 A。
- 在時間點 t_1 時，此時損益
  A : $F_1 - F_0$
  B : $-(F_1 - F_0)$

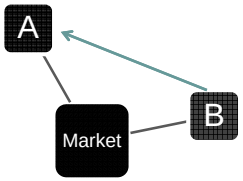
反向契約

時間軸

- 隨者時間前進現貨價格持續上漲。
- 時間點從 t_1 到 t_2 future 價格從 F1漲F2



買賣示意圖

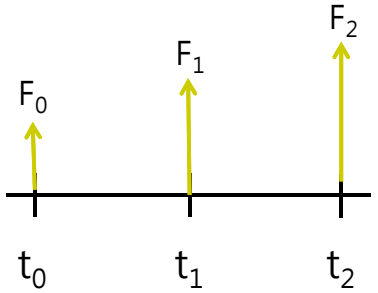


- 這短期間B的損失會繼續下去 。
- 在時間點 t_2 時，此時損益
  A : $F_2 - F_1$
  B : $-(F_2 - F_1)$

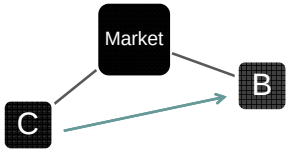
反向契約

時間軸

- B希望能夠減少損失，所以反向操作future。
- B在時間點從 t_1 時向C買進一口到期日為 t_2 future



買賣示意圖

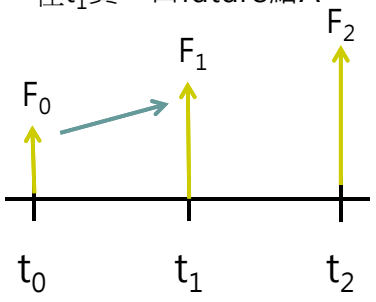


- 到了到期日 t_2 時，此時損益
  B : $F_2 - F_1$
  C : $-(F_2 - F_1)$

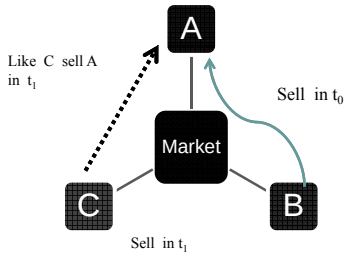
反向契約

時間軸

- 將t_0 到 t_2 的交易一起表示
- B在t_0賣的future損失將會被t_1買進的future獲利抵消
- 以現金流來看，可以看做C在t_1賣一口future給A



買賣示意圖



時間點/每期損益	A	B	C
t_1	$F_1 - F_0$	$-(F_1 - F_0)$	0
t_2(原本)	$F_2 - F_1$	$-(F_2 - F_1)$	0
t_2(向C買進)	0	$F_2 - F_1$	$-(F_2 - F_1)$

台灣的期貨市場

▷期指報價：台指期 小台指 電子期 金融期 台灣50期 元富預約開戶

台指期 (權值表) 資料日期：2004/10/13

名稱	時間	成交價	買進	賣出	漲跌	總量	基差	昨收	開盤	最高	最低	元富期貨下單
台指期10	10:07:51	6000	6000	6004	△9	10806	-33.72	5991	6020	6028	5985	買 賣 口數 送出
台指期11	10:07:49	5990	5990	5995	△10	605	-23.72	5980	5980	6022	5980	買 賣 口數 送出
台指期12	10:04:49	5990	5951	6025	△20	1	-23.72	5970	5990	5990	5990	買 賣 口數 送出
台指期03	08:50:22	5921	5921	5980	▽29	1	+66.23	5950	5921	5921	5921	買 賣 口數 送出
台指期06		—	5901	5950	—	—	—	5929	—	—	—	買 賣 口數 送出

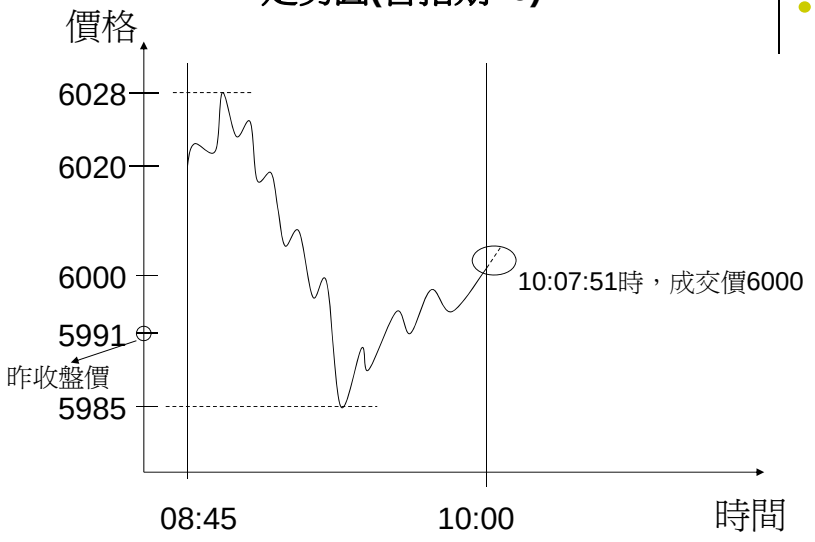
小台指 (權值表) 資料日期：2004/10/13

名稱	時間	成交價	買進	賣出	漲跌	總量	基差	昨收	開盤	最高	最低	元富期貨下單
小台指10	10:07:51	6000	5996	6005	△9	2021	-33.72	5991	6022	6025	5985	買 賣 口數 送出
小台指11	10:07:23	5988	5987	5998	△8	186	-21.72	5980	6000	6020	5982	買 賣 口數 送出
小台指12		—	5988	6014	—	—	—	5970	—	—	—	買 賣 口數 送出
小台指03	08:46:19	5947	5931	5989	▽3	3	+19.28	5950	5930	5947	5930	買 賣 口數 送出
小台指06	09:52:12	5938	5911	5947	△9	3	+41.86	5929	5922	5938	5922	買 賣 口數 送出

標的物: 股票指數

Check: http://tw.stock.yahoo.com/future/

走勢圖(台指期10)

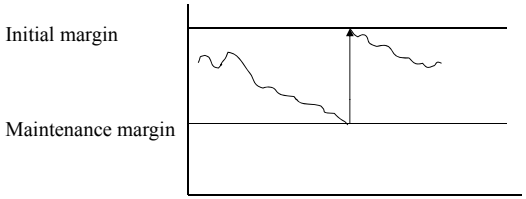


報價表(名詞解釋)

- 名稱：台指期10，指交割月份爲10月份的台指期貨。
  (期貨報價通常會揭露近三個連續近月與二個接續季月共五個資訊)
- 時間：10:07:51，指系統揭露成交的時間。
- 漲跌：△9，指成交價減去昨天收盤價的價差(6000-5991=9)。
- 總量：10806，指08:45~10:07期間市場成交的總數量。
- 基差：-33.72，指現貨價格減去期貨的價格，由此報價表可得知台股指數約爲5966點。(基差反應的是持有現貨的成本、收益與市場對未來走勢的預期心理)
- 最高：6028，指08:45~10:07期間內最高價。
- 平倉：以等量但相反買賣方向沖銷原有的部位。
- 未平倉量(open interest)：買（賣）的期貨契約在未平倉以前稱爲未平倉量，代表等量 的多頭部位與空頭部位。(報價表中常見的名詞)

保證金清算

- 替代原則:結算公司介入每筆交易，成爲買方的賣方，也成爲賣方的買方，承擔雙方之信用風險



- 保證金低至觸及維持保證金時，投資人會收到保證金催繳通知，必須將保證金補足至原始保證金水準，否則期貨商可逕自予以平倉

Example:期貨保證金清算

- 假設某投資人於2006年11月23日買進一口台股指數期貨契約，成交價格是5600點，原始保證金爲105,000元，維持保證金爲81,000元，此契約的總價值爲$1,120,000元(大台指契約乘數每點200元) 。在11月24、25、26日時，台指期收盤價爲分別爲5500、5450、5700點，期貨保證金清算過程如下:

Example:期貨保證金清算

日期	台指期收盤價	當日損益	保證金餘額
11/23	5600	—	105,000
11/24	5500	-20,000	85,000
11/25	5450	-10,000	75,000
11/26	5700	50,000	155,000

保證金餘額低於維持保證金$81,000，該投資人會接到經紀商通知(margin call)，並將保證金補足至原始保證金，故26日當天一開始的餘額變爲$105,000元。

Daily cash flows

- 假定第i天的future price爲f_i
- The contract cash flow at day i should be $f_i - f_{i-1}$
- Net cash flow:

$$(f_1 - f_0) + (f_2 - f_1) + (f_3 - f_2) + \ldots + (f_T - f_{T-1})$$
$$= f_T - f_0 = S_T - f_0 \qquad \because S_T = F_T$$

- It may differ because of the reinvestment and the margin system.

A futures contract has the similar accumulated payoff to a forward contract.

Relationships between Forward and Futures Prices

- Forward price = futures prices, if the interest rate is not stochastic.
- Let Forward price is **F** Futures price is **f**
- Consider forward and futures contracts on the same underlying asset with n days to maturity.
- The daily interest rate for day i is r_i.
- One dollar at the beginning of day i grows to $R_i \equiv e^{r_i}$ by day's end.

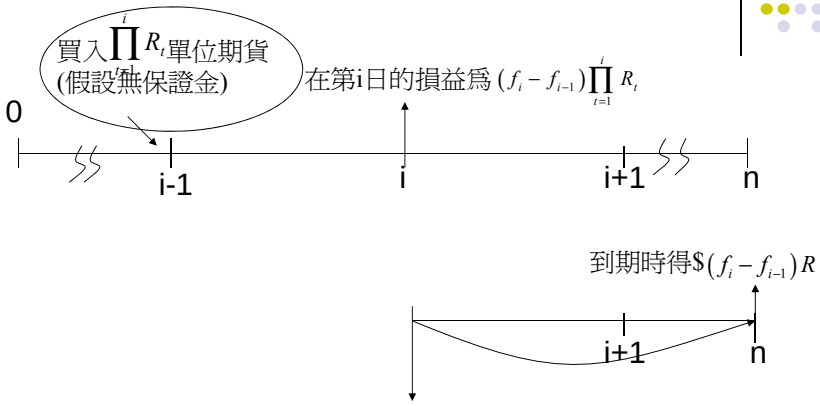
Proof

- Let fi be the futures price at the end of day i.
- So \$1 invested in the n-day discount bond at the end of day n will be worth \$**R**

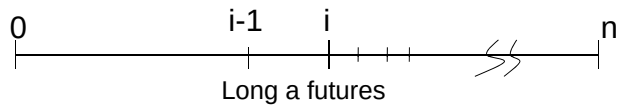
$$1 \times e^{r_1} e^{r_2} \ldots\ldots e^{r_n} = R_1 R_2 \ldots R_n = \prod_{t=1}^{n} R_t = R$$

- Consider the follow strategy:
  - Long $\prod_{t=1}^{i} R_t$ futures positions at the end of day $i-1$ and invest the cash flow at the end of day i in riskless bonds maturing on delivery day n.

Proof



$$\because (f_i - f_{i-1})\prod_{t=1}^{i} R_t \prod_{t=i+1}^{n} R_t = (f_i - f_{i-1})\prod_{t=1}^{n} R_t = (f_i - f_{i-1})R$$



Forwards 可視為n個futures所組成！

- The value at the end of day n is

$$\sum_{i=1}^{n} (f_i - f_{i-1})R = (f_n - f_0)R = (S_T - f_0)R$$

- Recall the value of R units of forwards is $(S_T - F_0)R$
- Arbitrage opportunity exists if $f_0 \neq F_0$.

Relationships between Forward and Future Prices

- 當利率隨機波動時, future price和forward price的理論價格不等
  一般而言,兩者差距不大
- Unless stated otherwise, assume forward and futures prices are identical.

Futures on Commodities

- For a commodity hold for investment purposes and with zero storage cost, the futures price is $F_0 = S_0 e^{rT}$
- In general, **U** stands for the PV of storage cost incurred during the life of a futures contract, and the futures price on commodity is $F_0 = (S_0 + U)e^{rT}$

$$\because F_0 = (S_0 - I)e^{rT} \quad I \text{ is the present value of income}$$
$$\because U \text{ is the present value of storage cost}, U = -I$$
$$\therefore F_o = (S_0 + U)e^{rT}$$

Homework

- You are provided the following information.
  Current price of wheat = $19,000 for 5000 bushels
  Riskless rate = 10 % (annualized)
  Cost of storage = $200 a year for 5000 bushels
  One-year futures contract price = $20,400 (for a contract for 5000 bushels)

(a) What is F* (the theoretical price)?

(b) How would you arbitrage the difference between F and F*? (Specify what you do now and at expiration and what your arbitrage profits will be.)

Futures on Commodities (convenience yield)

- Because user of the commodity may feel that ownership of the physical commodity provides benefit that not obtained by holders of futures contract.
- The benefit are the convenience yield (**y**) provided by the product. $F_0 e^{yT} = (S_0 + U)e^{rT}$
  - Ex: Oil refiner would like to take crude oil
- The convenience yield for investment asset=0 to prevent arbitrage.

Cost of Carry

- The relationship between futures prices and spot prices can be summarized in term of the cost of carry.
- This measures the dollar cost of carrying the asset over a period and consists of interest rate **r**, storage cost at the rate of **u**, minus cash flow at the rate of q generated by the asset.
  - The cost of carry is $C \equiv r+u-q$
- The futures price is $F_o = S_o e^{CT}$

Example

- A manufacturer needs to acquire gold in 3 months. The following options are open to her:(1) Buy the gold now. (2) Long one 3-month gold futures contract and take delivery in 3 months. If the T-bill are yielding an annually compound rate of 6%. What is the cost of carry for owning 100 ounces of gold at \$350 per ounce for a year?

Example (Solution)

- The cost of ownership is 6%, because it represents the interest one would have earned if one had bought a T-bill instead of the gold.
- The cost of carry owning 100 ounces for 1 year is C=100×\$350×0.06=\$2100.

The relationship cost of carry, convenience yield and basis

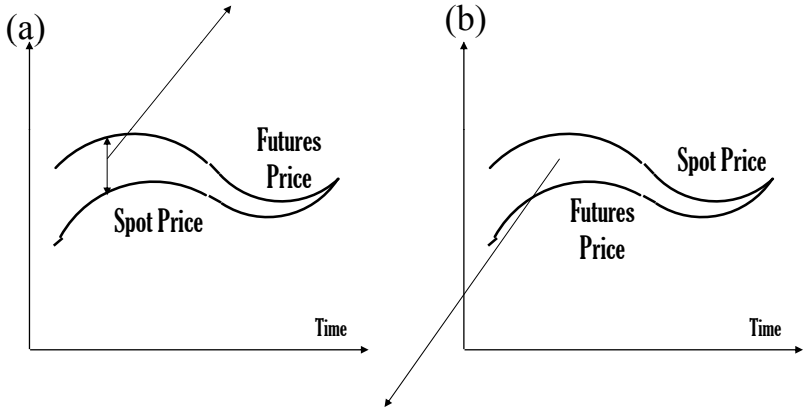
- **basis=$S_0 - F_0$**

$$F_o e^{yT} = S_o e^{CT} \rightarrow F_o = S_o e^{(C-y)T}$$

- In case of consumption assets, the futures price is greater than the spot price (basis $<$ 0) reflecting that the cost of carry.

Convergence of Futures Price to Spot Price

在正常市場下，基差(basis)為負，反應的就是標的資產的持有成本，隨著到期日接近基差就會越來越小。



思考:為何現貨價格會大於期貨價格(逆向市場)?

Hedging with Futures

- A long hedge is that involve taking a long position in a futures contract, when the hedger will have to purchase a certain asset in the future and wants to lock in a price now.
- A short hedge is that involve taking a short position in a futures contract, when the hedger owns an asset and expects to sell it at some time in the future.

(see example)

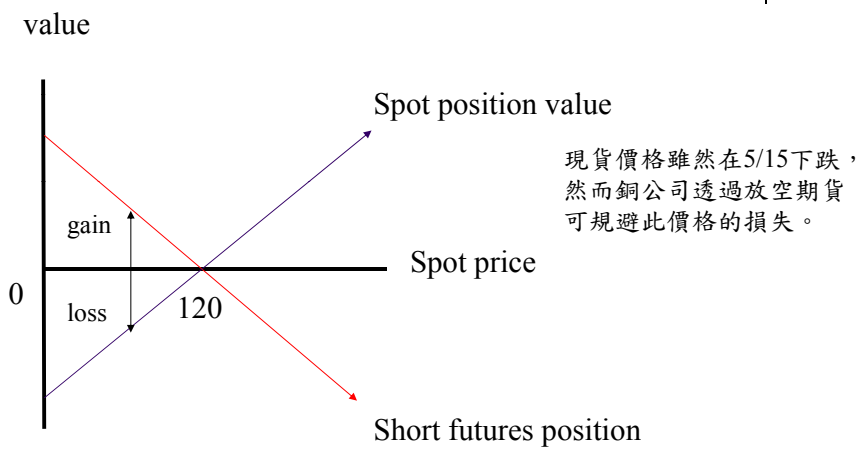
Example－Short Hedge

- 假設今天(1月25日)銅的現貨價格為每磅120美分，某一國外銅開採公司預計其將在5月15日賣出100,000磅銅，且在COMEX交易的五月份到期(到期日假設為5月15日)之銅期貨價格為每磅120美元(契約乘數)，其中每一口銅期貨合約大小為25,000磅。(假設不用繳保證金)
- 如果5月15日銅的現貨價格為每磅105美元，該國外銅開採公司如何利用COMEX(紐約商品期貨交易所)銅期貨合約來規避未來銅的價格風險？其淨收益為何?

該國外銅開採公司擔心未來銅價格下跌，導致生產收入減少，故它可以賣出**四口**COMEX銅期貨合約來避險。

Spot Market	Futures Market
1/25: 現貨100000磅 價值為 $120×100000=12,000,000	1/25: 賣出四口5月份到期之銅期貨
5/15:賣出現貨100000磅 現貨價值減少為 $105×100000=10,500,000	5/15: 結算四口銅期貨 結算的利得為$(120－105) ×4×25000=$1,500,000
Opportunity Loss = $12,000,000－10,500,000 =$1,500,000	Gain = $1,500,000

空頭避險損益圖



現貨價格雖然在5/15下跌，然而銅公司透過放空期貨可規避此價格的損失。

In class Exercise

- 如果到期時銅的現貨價格爲每磅125美分,請分析其現金流?
- 思考：假如市場上沒有想要避險現貨的標的資產之期貨?或是現貨的交易日與期貨的到期日不相同?該怎麼辦?

完全避險的條件

- 從事避險之期貨標的物要和現貨商品一模一樣。
- 到期日要等於避險沖銷日。
- 對應於要避險之現貨部位，所買賣的期貨口數爲某一整數。

期貨合約避險之合約選擇

- 交叉避險(Cross Hedge)：實務上要找到與現貨完全相同的標的資產之期貨合約非常困難，故乃以與現貨相關性高標的資產替代，此種策略稱爲交叉避險。
- 避險者使用期貨合約避險，其效果好壞決定於期貨價格和現貨價格相關性高低。因此避險者選擇期貨合約需考慮的因素有二：
  (1)期貨合約標的資產的選擇
  (2)期貨合約交割月份的選擇

最適期貨避險數量

- 決定最適的期貨合約數量有二種方法：
  (1)單純避險法(Naive Hedge Method)
  (2)最小變異數避險比率法(Minimum Variance Hedge Ratio Method)

單純避險法

- 又稱完全避險法(Perfect Hedge Method)，指避險者買進或賣出和欲避險之現貨部位金額相同，但部位相反的期貨合約。
- 假設基差風險不存在，亦即現貨價格和期貨價格之變化是完全一致。
- **公式：**期貨合約口數=欲避險之現貨部位金額÷每口期貨合約價值。
- 實際上現貨價格的變化和期貨價格的變化未必會完全一致，因此單純避險法並不一定能將現貨價格風險完全規避。

Example

- 假設某一基金經理人持有價值新台幣20億的股票，而台股指數期貨目前價格為5000點，其每一點值新台幣200元，那麼該基金經理人為了防止其現股價格下跌的風險，他應該出售多少口台股指數期貨來避險？

每口台股指數期貨價值 = 5000 ×200 = 1,000,000

應出售的期貨合約口數 = 2,000,000,000 ÷1,000,000= 2,000 (口)

最小變異數避險比率法

- **定義**：找出使避險投資組合風險(變異數)最小的避險比率的方法。又稱為迴歸分析法。
- **假設**：
  △S：避險期間內現貨價格的變動
  △F：避險期間內期貨價格的變動
  σ_s：現貨價格變動的標準差
  σ_F：期貨價格變動的標準差
  ρ：△S和△F間的相關係數
- **公式**：
  h*(最小變異數避險比率) = $\rho \dfrac{\sigma_s}{\sigma_F}$

最小變異數避險比率

- 如果ρ=1且$\sigma_F = \sigma_s$，則$h^*=1$。(此時最小變異數避險比率=單純避險法之避險比率，因爲期貨合約價格的變動和現貨價格的變動完全一致。)
- 假設其他情況不變，若σ_s愈大或ρ愈大，則h愈大
- 假設其他情況不變，若σ_F愈大，則h愈小。

最小變異數避險比率的推導

- Suppose we expect to sell N_A units of an assets at time t_2 and to hedge at time t_1 by shorting futures contracts on N_F units of a similar asset.
  —The hedge ratio is $h = \frac{N_F}{N_A}$
- S_1 and S_2 are the asset prices at time t_1 and t_2, and F_1 and F_2 are the futures prices at time t_1 and t_2.

最小變異數避險比率的推導

- Total amount realized for the asset denoted by Y

$$Y = S_2 N_A - (F_2 - F_1) N_F$$

$$\Rightarrow Y = S_1 N_A + (S_2 - S_1) N_A - (F_2 - F_1) N_F = S_1 N_A + N_A (\Delta S - h \Delta F)$$

- When the variance of △S—h△F is minimized, the variance of Y is minimized.

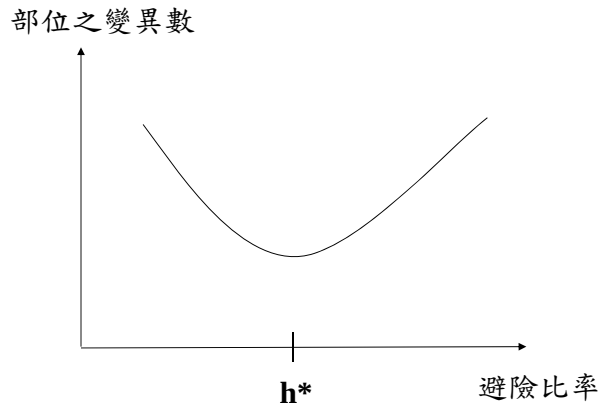
$$\because Var(\Delta S - h\Delta F) = \sigma_S^2 + h^2 \sigma_F^2 - 2h\rho\sigma_S\sigma_F = (h\sigma_F - \rho\sigma_S)^2 + \sigma_S^2 - \rho^2\sigma_S^2$$

$$\because MinVar(\Delta S - h\Delta F)$$

$$\Rightarrow (h\sigma_F - \rho\sigma_S)^2 = 0$$

$$\Rightarrow h = \rho \frac{\sigma_S}{\sigma_F}$$

避險者的部位之變異數和避險比率之關係



Example

- 假設台股指數價格變動百分比和大台指期貨價格變動百分比的相關係數爲0.8，而台股指數價格變動百分比之標準差爲0.6，而期貨合約價格變動百分比之標準差爲0.4，某一基金經理人持有價值新台幣20億的股票，而台股指數期貨目前價格爲5000點，其每一點值新台幣200元，那麼該基金經理人爲了防止其現股價格下跌的風險，他應該出售多少口台股指數期貨來避險？

Ans: 最小避險變異數比率爲0.8×(0.6÷0.4)=1.2

應出售的口數爲2000×1.2=2400(口)