# Realised tax benefits and capital structure

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**Abstract:** We examine the change of levered firm's capital structures due to different investment decisions of realised tax benefits and various sources of fund to finance coupon and dividend payouts. The complexity is analytically intractable but numerical approaches provide insights. Retaining realised tax benefits and investing them in risk-free assets instead of risky ones result in higher debt capacity and optimal firm value. The impact of positive-net-worth bond covenants on shareholders' investment decisions of realised tax benefits and the related agency problem are analysed. The impact of selling firm's asset (to finance payout) on optimal levered firm value is also analysed.

**Keywords:** credit risk; option pricing; realised tax benefits; capital structure.

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## 1 Introduction

In the option valuation framework for pricing credit risk, pioneered by Merton (1974) and Black and Cox (1976), asset values of levered firms evolve stochastically and default occurs when asset values fall below certain pre-specified boundaries. Brennan and Schwartz (1978) provide the first quantitative analysis of capital structure in a structural credit risk model with an exogenous fixed constant default boundary.[1] The market value of a levered firm ($V_L$) is the asset value ($V_A$) of an unlevered but otherwise identical firm adjusted for the present value of future tax benefits (TB) and bankruptcy costs (BC) associated with the use of debt.

$$\begin{aligned} V_L &= \mathrm{D} + \mathrm{E} \\ &= V_A + \mathrm{TB} - \mathrm{BC}. \end{aligned} \tag{1}$$

As the firm's asset value progresses, changes in the relative values of debt (D) and equity (E) that result from changes in the present value of future tax benefits (TB) and bankruptcy costs (BC) reflect the dynamics of optimal leverage. Leland (1994) argues, however, that the assumption of an exogenous default boundary is atypical, especially for long term debt (Smith and Warner, 1979). Moreover, because numerical analysis necessitates a finite debt maturity, the impact of an optimal fixed leverage policy on the levered firm value is not feasible in a Brennan and Schwartz (1978) framework.

Leland (1994) assumes that the only debt of the firm is a consol bond, and further, that the firm is restricted from selling assets (due to protective bond covenants) for financing coupon interest and dividend payments.[2] The 'no-asset-sale' constraint (Lando, 2004) allows Leland (1994) to derive time-homogeneous partial differential equations for debt value, interest tax shield, and bankruptcy cost as a function of the asset value ($V_A$). The default boundary is defined endogenously – default occurs when equity shareholders fail to raise sufficient equity capital from the sale of equity shares to meet current after-tax coupon interest payments. This model is widely accepted and extended (Leland and Toft, 1996; Broadie and Detemple, 1996; Briys and De Varenne, 1997; Hilberink and Rogers, 2002; Acharya and Carpenter, 2002; Fan and Sundaresan, 2000). Besides, Leland (1994) also compares the effects of endogenous and exogenous default boundaries on equity and debt values and shows that an exogenous default boundary alleviates the agency problem; namely that, equity shareholders cannot increase equity value at the expense of bondholders by making firm's activities more risky.

Leland (1994) model restricts the firm from selling the asset to finance the coupon payments. He provides a simplified and intuitive argument to show that the coupon payments are partially offset by the tax reduction and the remaining payments are financed by issuing equities. Indeed, the tax refund would be received by the firm as part of its retain earnings rather than directly paying to the equity holders (to offset the coupon payments). Fan and Sundaresan (2000) also argues that the retain earnings are accessible by the debt holders upon bankruptcy. This paper merges Fan and Sundaresan's argument into Leland's model by analysing the effect of different strategies to invest realised tax benefits.

Analysing the effect of retaining realised tax benefits is a complicated problem. On the one hand, retaining interest tax shields increases the levered firm's asset value which decreases the likelihood of default. On the other hand, the higher pre-tax coupon interest payments by equity shareholders raise the endogenous default boundary, and thereby, increase the likelihood of default. We will show that the former dominates the latter effect when the coupon interest payment (or leverage ratio) is low. Taking realised tax benefits into account increases levered firm value, and thereby, debt value. The converse is true when the coupon payment (or leverage ratio) is high.

To model the two offsetting effects carefully, it is necessary to model the accumulation process for realised tax benefits. The process depends on how realised tax benefits are invested by the firm. In an aggressive 'scalable investment' strategy, levered firms invest all realised tax benefits, $\tau C$, in the same risky asset pools, where $\tau$ denotes the tax rate. The levered firm's assets increase to $V_A + \tau C$ from $V_A$, and assuming constant returns to scale, all future earnings of the levered firm increase by $\tau C$ / $V_A$. In a conservative 'liquidity' strategy, levered firms invest all realised tax benefits in a risk-free asset, and the cash balance, provides liquid reserves against unexpected contingencies. The accumulated realised tax benefits grow at the risk-free rate between two coupon interest payment dates and jumps up by a fixed amount $\tau C$ at each coupon interest payment date. Our results show that compared to the liquidity strategy, the scalable investment strategy increases the overall risk of the firm, and consequently, increases equity values. Further, we confirm that an exogenous default boundary can alleviate the agency problem because equity shareholders cannot increase equity value at the expense of bondholders by investing realised tax benefits in risky assets.

The effect of realised tax benefits on credit risk and optimal capital structure cannot be evaluated analytically and must be estimated numerically. In particular, the stochastic process that describes the levered firm's asset value accounting for realised tax benefits following a scalable investment strategy can be modelled by a stair tree model for pricing stock options with fixed dividend payments (Dai, 2009). The down jump in the stock price due to a fixed dividend payment is replaced by an up jump in the firm's asset value due to a realised tax benefit. The robustness of stair tree is verified in Table 1 by observing that the tree accurately approximates the analytical formulas in Leland (1994) model by setting the finite maturity of its corporate debt $T$ to a large number.[3]

**Table 1** The impact of realised tax benefits

| $\tau$ | $C$ | *Leland (1994)* | | | | | | *Scalable investment strategy* | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | *Formula* | | | *Tree* | | | *Tree* | | |
| | | *Equity* | *Debt* | *Firm* | *Equity* | *Debt* | *Firm* | *Equity* | *Debt* | *Firm* |
| 0.35 | 7 | 27.655 | 100.435 | 128.090 | 27.679 | 100.439 | 128.109 | 26.235 | 104.242 | 130.477 |
| | 8 | 19.284 | 105.642 | 124.926 | 19.307 | 105.623 | 124.387 | 17.246 | 108.890 | 126.136 |
| 0.45 | 7 | 37.621 | 106.345 | 143.966 | 37.640 | 106.396 | 144.036 | 36.520 | 111.063 | 147.583 |
| | 8 | 29.717 | 115.725 | 145.442 | 29.740 | 115.793 | 145.533 | 27.992 | 121.876 | 149.868 |

Notes: The first column shows the tax rate $\tau$, and the second column, the coupon interest payment $C$ parameterisations. In Leland (1994) model, realised tax benefits are implicitly transferred to equity shareholders. Under scalable investment strategy, levered firms invest realised tax benefits in the same risky asset pools. Formula and tree denote that the values are priced by analytical formulas and by the discrete-time stair tree models with $T = 200$ years, respectively.

Table 1 also illustrates the impact on debt, equity, and levered firm values for levered firms with consol corporate debt that adopt a scalable investment strategy and maintain a fixed leverage. When realised tax benefits are retained by the levered firm, equity values under a scalable investment strategy are always lower than equity values when realised tax benefits immediately accrue to equity shareholders as in Leland (1994). In the table, the retention of realised tax benefits by the firm decreases the likelihood of default and increases the collateral available for debt. Consequently, both debt and levered firm values are higher under a scalable investment strategy. Furthermore, if the firm issues more equity than is required for coupons, then the remaining proceeds would be invested as a part of the firm's asset, which becomes accessible by the debt holders upon bankruptcy. Take the numerical experiment with $C = 7$ and $\tau = 0.35$ in Table 1 for example. The firm may issue more equity, say $2C$, and then the remaining proceeds ($\tau C + C$) are invested back to the firm's asset pool (i.e., scalable investment strategy). This would increase debt value from 104.242 to 112.222 since the sole increment of the firm's asset without changing other factors would reduce the default risk.

Indeed, our numerical model is flexible enough for more general specifications that cannot be solved analytically. For example, it is more typical that firms are allowed to sell assets to finance coupon interest or dividend payments. And to keep the problem analytically tractable, we can as in Leland (1994), assume that when asset sales (Lando, 2004) are allowed, cash proceeds are proportional to the firm's asset value.[4] The impact on debt and equity can then be precisely accounted as follows. When cash proceeding from asset sales exceeds coupon interest payments, excess cash flows to equity shareholders as a special dividend. When cash proceeding from asset sales are insufficient to meet the coupon interest payments, then equity shareholders finance the remainder of coupon interest payments through new shares issuance. Alternatively, as in Merton (1974) and Brennan and Schwartz (1978), we can also numerically evaluate the analytically intractable case when coupon interest payments are completely financed through asset sales – call it total 'asset-sale' assumption for simplicity.

Our numerical results analyse the impacts of different asset-sale assumptions on optimal leverage ratios of levered firms. Comparing to 'no-asset-sale' assumption, the proportional 'asset-sale' assumption decreases the debt capacity and optimal leverage. Under these two assumptions, the firm defaults if the equity shareholders fail to raise sufficient equity capital to meet the payments due that is not covered by selling the firm's asset, if any. However, under the total 'asset-sale' assumption, equity shareholders do not need to repay any payments due and the firm defaults only when its asset value fails to cover payments due. This change of the default condition results in much higher debt capacity and the optimal levered firm value than the other two asset-sale assumptions.

The paper is organised as follows. In Section 2, we first describe the mathematical model, the 'asset-sale' and 'no-asset-sale' assumptions made, as well as the exogenous/endogenous default boundaries described in Leland (1994). We then show how the aforementioned models are adjusted to reflect realised tax benefits and the numerical method used for evaluating these models. Section 3 considers how realised tax benefits affect optimal leverage under the 'no-asset-sales' constraint and how the existence of positive net-worth bond covenants influence equity shareholder decisions on how realised tax benefits are invested. Section 4 extends the discussion to the 'asset-sale' case. We show that while both the debt capacity and optimal leverage are decreased under the proportional asset-sale assumption, higher levered firm and debt capacity can be achieved when there are no bondholder protective covenants – for example, a restriction on asset sales or positive net-worth pledge. Section 5 concludes the paper.

## 2 Preliminaries

In Merton (1974), Brennan and Schwartz (1978), and Leland (1994), levered firm asset value $V_A$ follows a (geometric) diffusion process:

$$dV_A = \left(\mu V_A - P\right)dt + \sigma V_A dz, \tag{2}$$

where $\mu$ is the instantaneous expected rate of return on levered firm assets; $P$ is the payout to finance coupon interest and dividend payments; $\sigma$ is the volatility of levered firm's asset returns; $dz$ is a Winner process. When no-asset-sales are allowed, $P$ is zero. When as in Merton (1974) and Brennan and Schwartz (1978), asset-sales are permitted and coupon interest and dividend payments due is completely financed through asset sales; i.e., $P$ is equal to the sum of instantaneous dividend and coupon interest payments. And as in Leland (1994), proportional asset-sales are assumed for analytical tractability; $P \equiv DV_A$, where the proportion $D$ is defined by the coupon interest payment, that is, $D = (1 - \tau)C / V_A(0)$. Note that $P$ can be greater or less than the coupon interest payment as $V_A$ evolves over time. In Leland (1994), the payout $P - (1 - \tau)C$ becomes a special dividend payment when $P$ exceeds the after-tax coupon interest payment $(1 - \tau)C$; any after-tax coupon interest payment $(1 - \tau)C - P$ deficits are financed from the sale of additional equity shares.

Default is triggered when a firm's asset value falls below thresholds called default boundaries. The boundary can be exogenously defined as a function of firm's liability structure, e.g., as a constant proportion of the debt face value (Nielsen et al., 2001; Kim et al., 1993; Longstaff and Schwartz, 1995), or as the discounted present value of the debt (Black and Cox, 1976). The default boundary can also be endogenously defined. For example, a default event can be triggered when the payout $DV_A$ cannot meet coupon interest payments as in Kim et al. (1993), or when the shareholders fail to raise sufficient equity capital to meet after-tax coupon interest payments as in Leland (1994).

The realised tax benefit is the reduction in tax liability from making a coupon interest payment. Retaining the realised tax benefit changes the amount the equity shareholders should raise to finance the interest payments and the process of the firm's asset value. Take the no-asset-sale assumption for example, equity shareholders should sell additional equity shares to finance the total coupon interest payment $C$ instead of the after-tax coupon interest payment $(1 - \tau)C$. Besides, the stochastic process for the firm's asset value also depends on the strategies employed for investing realised tax benefits.

When realised tax benefits are completely invested by levered firms in the same pool of risky assets instead of in a risk-free asset, future earnings increase in proportion to realised tax benefits. The stochastic process of the levered firm's asset value under a scalable investment strategy when coupon interest and dividend payments are financed by sale of equity shares is

$$dV_A = \left(\mu V_A + \tau C\right)dt + \sigma V_A dz, \tag{3}$$

and when levered firms sell a fixed proportion of its assets to finance coupon interest and dividend payments, is[5]

$$dV_A = \left((\mu - D)V_A + \tau C\right)dt + \sigma V_A dz. \tag{5}$$

In contrast, when realised tax benefits are fully invested in a risk-free asset, the levered firm's asset value under a liquidity strategy is the sum of $V_A$ and the accumulated value from investments of realised tax benefits in a risk-free asset, $V_A^R$, where

$$dV_A^R = \left(rV_A^R + \tau C\right)dt. \tag{6}$$

Note that the realised tax benefits are no longer invested by levered firms in the same pool of risky assets, the processes of $V_A$ under different asset-sale assumptions are obtained by removing the realised tax benefit term $\tau C$ from equations (3), (4), and (5).

The evolution of a levered firm's asset value is modelled as a discrete-time tree process in this paper. When realised tax benefits are not retained, this process can be trivially simulated by a CRR tree model (Cox et al., 1979). Otherwise, the realised tax benefit at each time step is $\tau C\Delta t$, where $\Delta t$ denotes the length of the time step. Under a scalable investment strategy, the stochastic process of asset value is simulated by a stair tree for pricing stock options with fixed divided payments (Dai, 2009), where the down jump of the stock price due to a dividend payment in the stair tree is replaced by the up jump of $\tau C\Delta t$ in the levered firm's asset value. Under a liquidity strategy, the evolution of $V_A^R$ contains no stochastic terms and is easily calculated.

Equity and debt, represent contingent claims on a levered firm's asset value, and its values, $\mathrm{E}(t, V_A)$ and $\mathrm{D}(t, V_A)$ are computed in a discrete-time tree model framework using backward induction. At the debt maturity, $T$, default occurs when the levered firm's asset value is less than the sum of debt's face value $F$ and after-tax coupon interest.

$$\mathrm{E}\left(T, V_A\right) = \begin{cases} V_A - F - (1-\tau)C\Delta t & \text{if } V_A \geq F + (1-\tau)C\Delta t \\ 0 & \text{otherwise} \end{cases},$$

and

$$\mathrm{D}\left(T, V_A\right) = \begin{cases} F + C\Delta t & \text{if } V_A \geq F + (1-\tau)C\Delta t \\ (1-\alpha)V_A & \text{otherwise} \end{cases},$$

where $(1 - \alpha)$ is the asset recovery rate (i.e., $\alpha$ is the bankruptcy cost). The impact of maintaining fixed leverage can be approximated in a finite debt maturity model by letting $T$ become large.

The backward induction is complex since the levered firm value at each discrete time step has to be adjusted for asset sales to finance the coupon interest payout as well as realised tax benefits. We use the superscripts '–' and '+' to denote the levered firm's asset values before and after adjustment, respectively. Note that the occurrence of an adjustment implies that the levered firm did not default at that time step. The value of any contingent claim on the after-adjustment asset value $V_A^+$ at time $t$, denoted as $f(t, V_A^+)$, can be expressed as the discounted expected value of the contingent claim at the next time step. In a binomial tree, we have

$$f\left(t, V_A^+\right) \equiv e^{-r\Delta T}\left(p \times f\left(t+\Delta t, \hat{V}_A^-\right) + (1-p) \times f\left(t+\Delta t, \check{V}_A^-\right)\right),$$

where the levered firm's asset value $V_A^+$ will move to asset values $\hat{V}_A^-$ and $\check{V}_A^-$ with probabilities $p$ and $1 - p$ at the next time step: $t + \Delta t$. The general backward induction formulas for equity and debt values are defined as

$$\mathrm{E}(t, V - A) = \begin{cases} \mathrm{E}\left(t, V_A^+\right) - S & \text{if default does not occur} \\ 0 & \text{if default occurs} \end{cases},$$

and

$$\mathrm{D}\left(t, V_A^-\right) = \begin{cases} \mathrm{D}\left(t, V_A^-\right) & \text{if default does not occur} \\ (1-\alpha)V_A^- & \text{if default occurs} \end{cases},$$

where $S$ denotes the payout from the sale of additional equity to finance the coupon interest payment. As in Leland (1994), default can be triggered endogenously by the inability of the firm to raise sufficient equity capital to meet the payout; i.e., $\mathrm{E}(t, V_A^+) < S$. Or as in Brennan and Schwartz (1978), default can be triggered exogenously when the firm's asset value falls below an exogenous default boundary $V_B$, i.e., $V_A^- < V_B$.

We benchmark our analysis to the *realised tax benefits not retained* Leland (1994) case. Under a no-asset-sale constraint, equity shareholders sell additional equity shares to finance after-tax coupon interest payments, i.e., $S = (1 - \tau)C\Delta t$, and as in Leland (1994), the levered firm's asset value does not change, i.e., $V_A^+ = V_A^-$. When the levered firm is allowed to sell a fixed proportion $D$ of its assets to finance coupon interest and dividend payments, then as in Leland (1994), $S = (1-\tau)C\Delta t - DV_A^-\Delta t$ and $V_A^+ = V_A^- - DV_A^-\Delta t$. Note that a negative $S$ implies that cash inflow from asset sales exceeds the coupon interest payment and excess proceeds go to equity shareholders as a special dividend. Lastly, when firms are allowed to sell assets to fully finance the coupon interest payment as in Merton (1974) and Brennan and Schwartz (1978), then $S = 0$ and $V_A^+ = V_A^- - (1-\tau)C\Delta t$.

When realised tax benefits $\tau\Delta C$ are retained by the firm, the payments from the sale of additional equity shares $S$ are higher than those in the *no realised tax benefits retained* Leland (1994) case by $\tau\Delta C$.[6] For example, under a no-asset-sale constraint, the payout for the coupon payment from the sale of additional equity shares becomes $S = (1 - \tau)C\Delta t + \tau C\Delta t = C\Delta t$. The adjustment on the firm's asset value depends on how realised tax benefits are invested. When realised tax benefits are invested in the same pool of risky assets, then $V_A^+$ are larger than those in the *no realised tax benefits retained* Leland (1994) case by $\tau C\Delta t$.[7] In contrast, when realised tax benefits are invested in a risk-free asset, the levered firm's asset is composed of a risky portfolio $V_A$ and a risk-free portfolio $V_A^R(t)$, which consists of riskless investments of the realised tax benefits, where $V_A^R(t)$ denotes the value of the riskless portfolio at time $t$. The settings of $V_A^+$ are the same as those in the *no realised tax benefits retained* Leland (1994) case.[8] The value of the riskless portfolio can be expressed as a function of time: $V_A^R(t) = e^{r\Delta t}V_A^R(t-\Delta t) + \tau C\Delta t$.

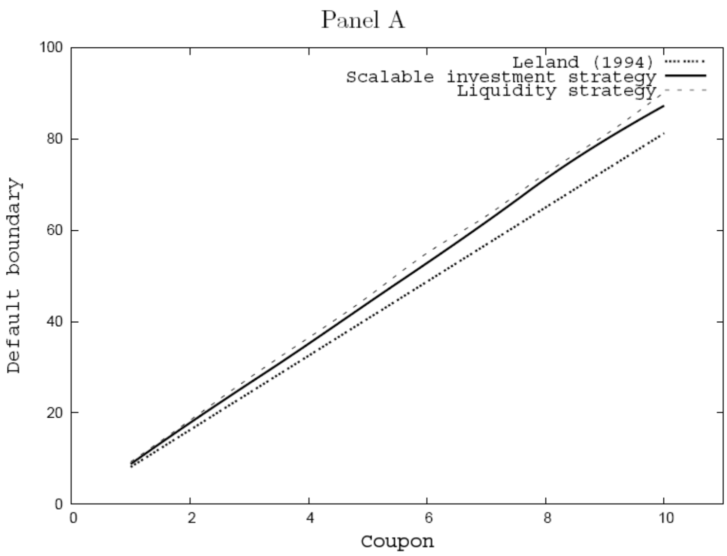
## 3 The impact of realised tax investment strategies under a no-asset-sale constraint

Levered firms are not allowed to sell assets to finance coupon interest and dividend payments under a no-asset-sale constraint. In Leland (1994), realised tax benefits implicitly flow to equity shareholders immediately. Alternatively, realised tax benefits can be retained by the levered firm and reinvested. The existence of exogenous default boundaries (due to positive net-worth bond pledges) affects equity shareholders' decisions about the strategies for investing and distributing realised tax benefits and optimal capital structure.

## *3.1 Endogenous default boundaries and equity values*

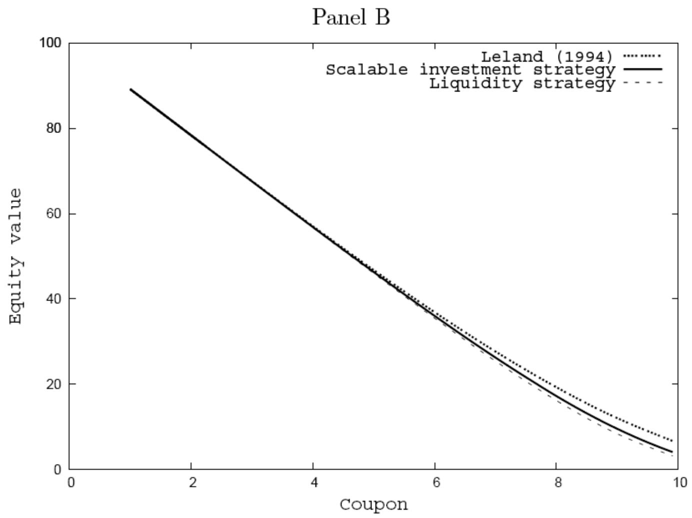
Under a no-asset-sale constraint, coupon interest and dividend payments cannot be financed through asset sales and these payments must be financed through new share issuance. Moreover, if levered firms are not restricted by exogenous default boundaries (due to protective net-worth bond covenants), defaults will only occur when the levered firm fails to meet payments due through new share issuance (i.e., when equity value is less than payments due). The endogenous default boundary $V_B$ denotes the threshold asset value when equity value is less than payments due. For strategies where the levered firm retains the realised tax benefits, equity shareholders at each time step issue new equity shares to finance the pre-tax coupon interest payment, $C\Delta t$, instead of the after-tax coupon payment $(1 - \tau)C\Delta t$. This increases $V_B$ and decreases equity values as illustrated in Figure 1.

**Figure 1** Default boundary and equity value under a no-asset-sale constraint




Notes: The dotted, solid, and dashed lines plot the default boundary and equity value when realised tax benefits are not retained as in Leland (1994) and when retained realised tax benefits are invested in risky assets following a scalable investment strategy, and in risk-free assets following a liquidity strategy, respectively. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%.

**Figure 1** Default boundary and equity value under a no-asset-sale constraint (continued)




Notes: The dotted, solid, and dashed lines plot the default boundary and equity value when realised tax benefits are not retained as in Leland (1994) and when retained realised tax benefits are invested in risky assets following a scalable investment strategy, and in risk-free assets following a liquidity strategy, respectively. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%.

Note that in Panel A, the default boundaries when realised tax benefits are retained are higher than when realised tax benefits are immediately distributed to equity shareholders as in Leland (1994). In Panel B, equity values when realised tax benefits are retained are lower than in Leland (1994) because realised tax benefits, which accrue only to equity shareholders in Leland (1994), are retained by the levered firm and accrue to bondholders when the levered firm defaults.

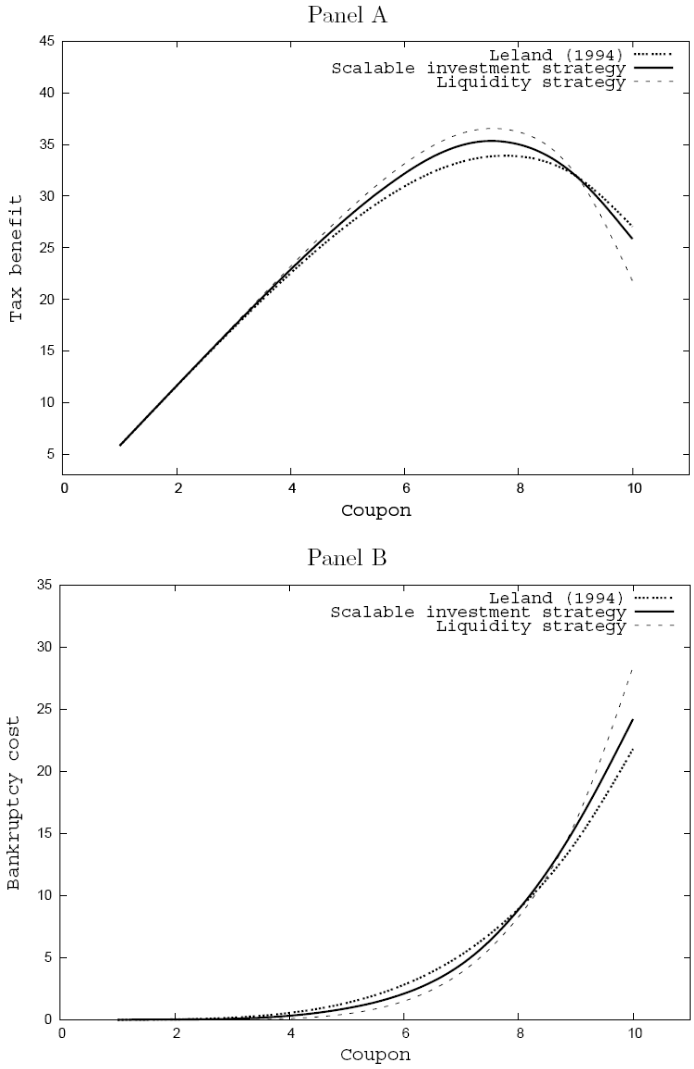
The alternative strategies for investing realised tax benefits also impact default boundaries and equity values. Recall that equity is a call option on the levered firm's asset and option value increases with the volatility of the underlying asset. The value of accumulated tax benefits and levered firm assets are more volatile in a scalable investment strategy that invests realised tax benefits in risky assets than in a liquidity strategy that invests realised tax benefits in a risk-free asset. So the equity value under a liquidity investment strategy is lower than the value under a scalable strategy given all other conditions are identical. Consequently, the default boundary under a liquidity investment strategy is higher than the boundary under a scalable investment strategy because a higher levered firm's asset value is needed under a liquidity investment strategy to ensure that equity value will be sufficient to meet payments due.[9]

## *3.2 Tax benefits and bankruptcy costs*

Retaining realised tax benefits increases levered firm's asset value that decreases the likelihood of default but also raises the default boundary that makes default more likely.

The offsetting effects on the present values of future tax benefits and future bankruptcy costs depend on the magnitude of the coupon interest payment.

**Figure 2** Tax benefits and bankruptcy costs under a no-asset-sale constraint




Notes: The dotted, solid, and dashed lines plot the present values of future tax benefits and future bankruptcy costs when realised tax benefits are not retained as in Leland (1994) and when retained realised tax benefits are invested in risky assets following a scalable investment strategy, and in risk-free assets following a liquidity strategy, respectively. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%.

From Figure 2, observe that when coupon interest payments are low, the increase in levered firm's asset value, which mitigates the likelihood of default, increases the present value of future tax benefits and reduces the present value of future bankruptcy costs. But when coupon interest payments are high, the higher default boundary, which increases in likelihood of default, decreases the present value of future tax benefits and raises the present value of future bankruptcy costs. We can infer that the effect of increase in levered asset value dominates the effect of higher default boundary when the coupon interest payment is relatively low, and the converse is true when the coupon interest payment is relatively high.

The alternative strategies for investing realised tax benefits also impact the present values of future tax benefits and future bankruptcy costs. When the coupon interest payment is relatively low, the levered firm is less likely to default and increasing the risk of the firm will greatly increase the likelihood of default. That is why the present value of future tax benefits is lower, and the present value of future bankruptcy costs is higher, under the more risky scalable investment strategy than those under the less risky liquidity strategy. In contrast, when the coupon interest payment is relatively high, the levered firm is more likely to default and increasing the risk of the firm will increase the likelihood of firm's survival.[10] Then the present value of future tax benefits is higher, and the present value of future bankruptcy costs is lower under the scalable investment strategy than those under the liquidity strategy.

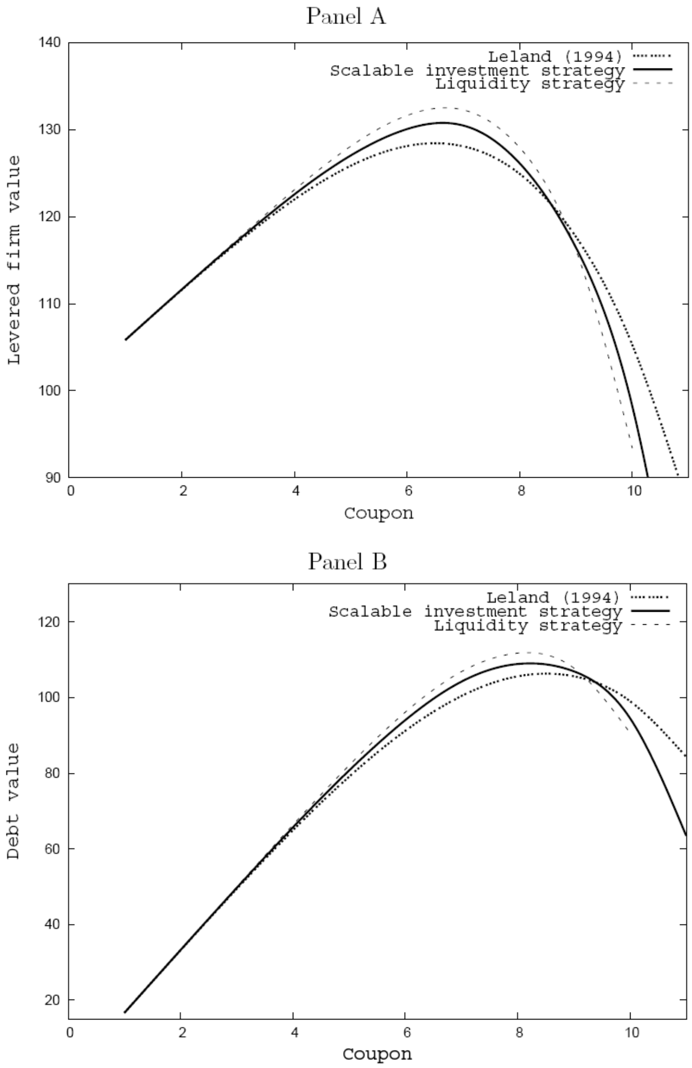
### *3.3 Levered firm values and debt values*

Figure 3 illustrates the effect on debt and levered firm values from the retention of tax benefits and the manner in which retained tax benefits are invested. Panel A of Figure 3 shows that when coupon interest payments are low, higher levered firm's asset values that decrease the likelihood of default dominate the rise in default boundary that makes default more likely. Levered firm values are higher when tax benefits are retained by the levered firm than immediately distributed to equity shareholders as in Leland (1994). But when coupon interest payments are high, the rise in default boundary that makes default more likely dominates higher levered firm's asset values that reduce the likelihood of default. Levered firm values are lower when tax benefits are retained than immediately distributed to equity shareholders as in Leland (1994).

Further, because as noted previously, default is less likely when coupon interest payments are low, levered firm value is relatively higher under a liquidity than scalable investment strategy. But when coupon interest payments are high, default is more likely. In this case, levered firm values are relatively higher under a scalable investment than liquidity strategy.

It can be observed from Figure 3 that both the firm value and the debt value increase with the coupon interest payment when the coupon is low. But they decrease with the coupon interest payment due to the significant increment of default risk when the coupon is high. Moreover, there is an optimal coupon interest payment that maximises debt values. A maximum debt value of \$106.4 occurs at a coupon interest payment of \$8.5, when as in Leland (1994), realised tax benefits are immediately distributed to equity shareholders. Higher collateral values from the retention of realised tax benefits increases debt capacity; namely, a higher maximum debt value of \$111.9 under the liquidity strategy and \$111.6 under the scalable investment strategy both increase debt capacity by approximately 5% (e.g., (111.9106.4) / 106.4 = 5.2%). In addition, the debt capacity under a liquidity strategy is slightly higher than the capacity under a scalable investment strategy. This arises because the value of a levered firm's assets is more volatile when retained realised tax benefits are invested in risky assets than in risk-free assets. As Leland (1994) notes, debt capacity decreases as the volatility of the levered firm's asset increases.

**Figure 3** Debt and levered firm values under a no-asset-sale constraint




Notes: The dotted, solid, and dashed lines plot debt and levered firm values when realised tax benefits are not retained as in Leland (1994) and when retained realised tax benefits are invested in risky assets following a scalable investment strategy, and in risk-free assets following a liquidity strategy, respectively. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%.

Compared to a maximum levered firm value of \$128.4 under the Leland (1994) case, the retention of realised tax benefits increases maximum levered firm value to \$130.6 under the scalable investment strategy and \$132.5 under the liquidity strategy. Maximum levered firm value is lower under the scalable investment strategy than liquidity strategy because the value of a levered firm's asset is more volatile when retained realised tax benefits are invested in risky than risk-free assets. Further, when coupon interest payments are low (high), the incremental increase (decrease) in levered firm value is small (large) relative to the incremental increase (decrease) in debt value because the retention of realised tax benefits also reduces equity values as illustrated in Panel B of Figure 1.

## 3.4 Protected versus unprotected debt

Our analysis has, thus far, focused on default triggered endogenously when the levered firm fails to raise sufficient equity capital to meet the coupon interest payment. Leland (1994) refers to this case as unprotected debt. We now consider the case when there is a positive net-worth covenant. In this case, default on protected debt occurs when the levered firm's asset falls below an exogenous default boundary.

Observe from Table 2, that levered firm values for protected debt are lower than unprotected debt. Levered firms gain more from leverage when debt is unprotected because, as Leland (1994) argues, equity shareholders can increase their value at the expense of bondholders by increasing the risk (volatility) of the levered firm's assets when debt is unprotected. Equity value increases from \$26.2 to \$52.8, and debt value falls, from \$137.5 to \$72.6, when the asset volatility increases from 20% to 60% under the scalable investment strategy. The results are similar under the liquidity strategy.

**Table 2** Values of protected and unprotected debt and equity for different levels of risk

| | *Volatility (%)* | *Unprotected debt* | | | *Protected debt* | | |
|---|---|---|---|---|---|---|---|
| | | *Equity* | *Debt* | *Firm* | *Equity* | *Debt* | *Firm* |
| Scalable investment strategy | 20% | 26.2 | 137.5 | 163.7 | 16.6 | 87.2 | 103.8 |
| | 40% | 38.4 | 99.7 | 138.1 | 12.2 | 55.0 | 67.2 |
| | 60% | 52.8 | 72.6 | 125.4 | 8.7 | 49.0 | 57.7 |
| Liquidity strategy | 20% | 25.6 | 141.8 | 167.4 | 17.7 | 94.5 | 112.2 |
| | 40% | 32.1 | 107.1 | 139.2 | 13.5 | 59.7 | 73.2 |
| | 60% | 41.6 | 81.0 | 122.6 | 8.8 | 50.1 | 58.9 |

Notes: This table compares the values of debt and equity for both unprotected and protected debt under different tax benefit investment assumptions. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 50%, and the coupon interest payment is \$9.

As Jensen and Meckling (1976) point out, in a rational expectations equilibrium, the expected cost to bondholders will be passed back to equity shareholders through lower prices on newly issued debt. The imposition of a positive net-worth covenant avoids the agency problem and explains why protected debt is still found in real world markets. In the 'protected debt' column of Table 2, equity values decrease when the asset volatility increases regardless of how retained realised tax benefits are invested. With protected debt, equity shareholders do not have an incentive to increase firm risk at bondholders' expense. Similar arguments can be applied to analyse the investment policies of realised tax benefits. For unprotected debt, equity values are higher and debt values are lower under a scalable investment strategy than under a liquidity strategy. But for protected debt, investing realised tax benefits in risky assets lowers both equity and debt values. Thus the equity holders have no incentive to invest the realised tax benefits riskily with protected debt.

## 4 Analysis with asset sales

Now consider the scenario where debt is unprotected and levered firms are allowed to sell assets to finance coupon and dividend payments. Under a total sales scenario, coupon interest payments are fully financed by selling assets. Under a proportional sales scenario, coupon interest payments are partially rather than totally financed by asset sales. For analytical tractability, we assume, as in Leland (1994), which the predetermined proportion of assets sold $D$ is $(1 - \tau)C / V_A(0)$. Additional equity is raised only when asset sales are insufficient to fund the entire coupon interest payment and default occurs when the requisite equity capital cannot be obtained.

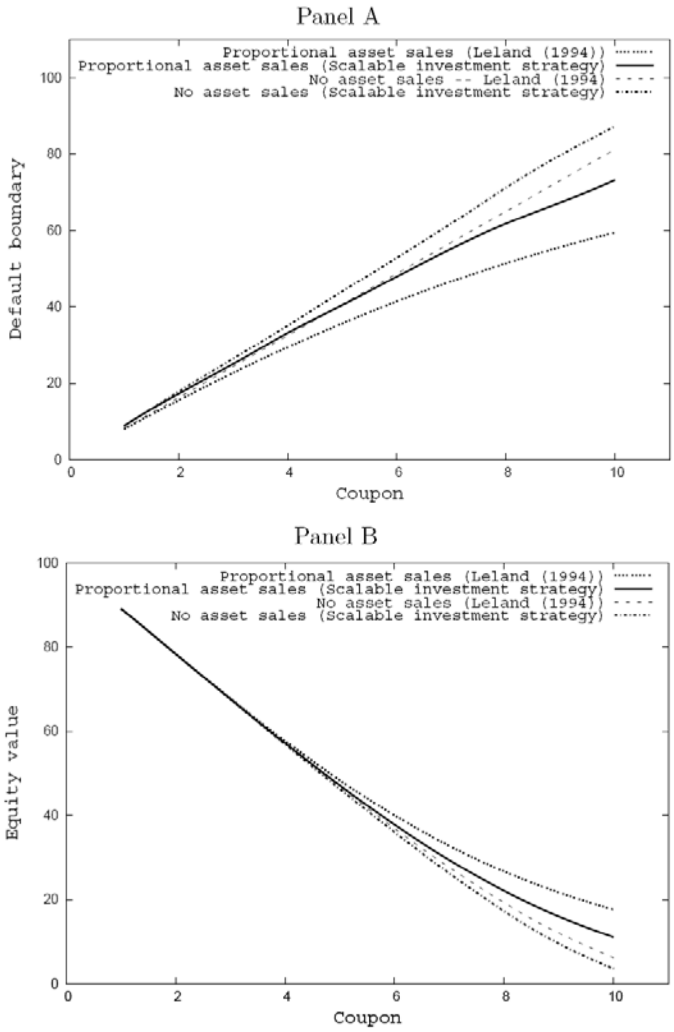
The subsequent analysis shows how an asset sales option influences equity, debt, and levered firm values. For simplicity, we assume that the retained realised tax benefits are invested in the same pool of risky assets.

### *4.1 Proportional asset sales*

Allowing for asset sales impacts default boundaries and equity values. In Panel A of Figure 4, proportional asset sales lower the default boundary significantly. This should not be surprising. When proportional asset sales are permitted, equity shareholders only need to raise additional equity when asset sales are insufficient to finance the coupon interest payment. Because the obligation for equity shareholders to repay the coupon interest payment is sharply reduced, equity values under proportional asset sales are higher than under a no-asset-sale constraint as shown in Panel B of Figure 4. The effects of retaining the realised tax benefits under proportional asset sales are similar to the effects under no-asset-sale constraint. The default boundary (equity value) under scalable investment strategy is higher (lower) than the boundary (equity value, respectively) under the *realised tax benefits not retained* Leland (1994) case since the former strategy needs to raise more equity capital to finance coupon payments then the latter one as mentioned in Section 3.1.

The impact of asset sales on debt and levered firm values is more complex. Asset sales to finance coupon interest payments reduce the levered firm's asset value and increase the likelihood of default. But asset sales also lower the default boundary and decreases the likelihood of default.

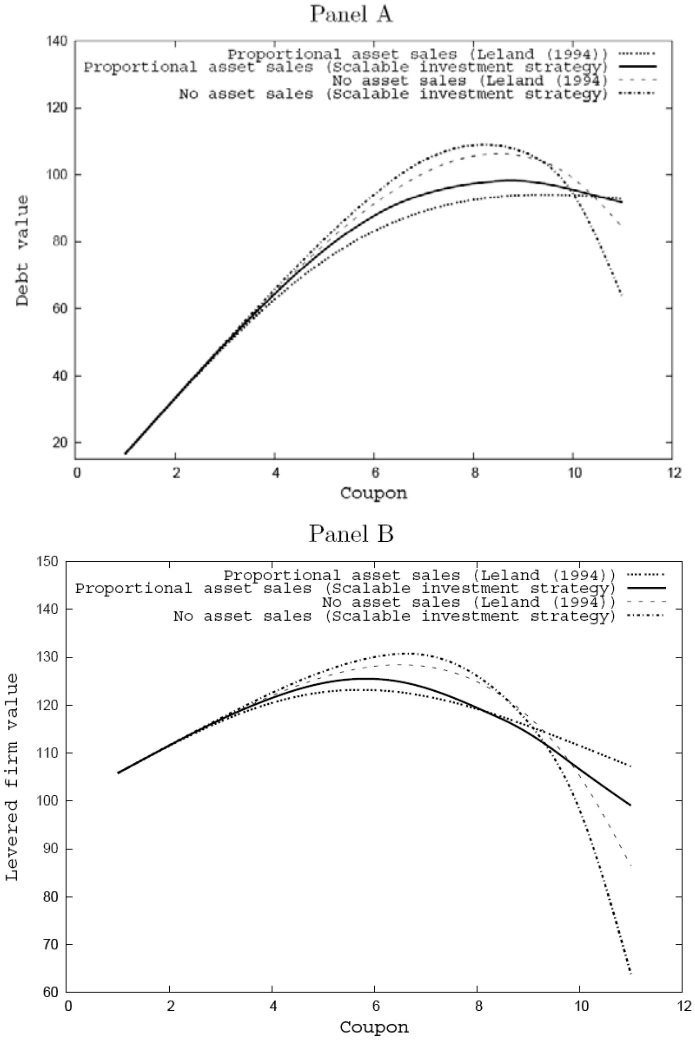
**Figure 4** The impact of proportional and no asset sales on default boundaries and equity values




Notes: The figures plot the impact of proportional and no asset sales on default boundary and equity value when realised tax benefits are not retained as in Leland (1994) and when retained realised tax benefits are invested in risky assets following a scalable investment strategy. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%.

As Figure 5 shows, the reduction in asset value dominates a lowered default boundary when coupon interest payments are relatively low. Debt and levered firm values under proportional asset sales are lower than under a no-asset-sale constraint. When coupon interest payments are relatively high, a lowered default boundary dominates the reduction in asset value. Debt and levered firm values under proportional asset sales are higher than under a no-asset-sale constraint. Similarly, when the coupon is relatively low, both the debt and the levered firm values under scalable investment strategy are higher than those under no realised tax benefits retained Leland case. And the reverse is true when the coupon is relatively high.

**Figure 5** The impact of proportional and no asset sales on debt and levered firm values




Notes: The figures plot the impact of proportional and no asset sales on debt and levered firm values when realised tax benefits are not retained as in Leland (1994) and when retained realised tax benefits are invested in risky assets following a scalable investment strategy. The levered firm's asset value is initialised at $100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%.

Although restricting levered firm from selling its asset to finance coupon interest payments reduces the equity value as illustrated in Panel B of Figure 4, it increases the debt capacity and the maximal firm value as in Figure 5. This is because that a no-asset-sale constraint provides protections for bond holders but limitations for equity shareholders. However, even if issuing corporate debts with a no-asset-sale constraint reduces the equity value, equity shareholders still have incentive to do so to benefit from the increments of debt capacity and maximal firm value.

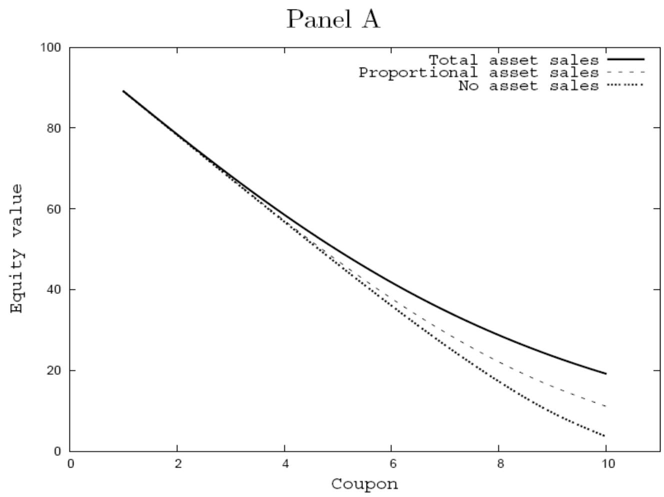
### 4.2 Total asset sales

As Merton (1974) and Brennan and Schwartz (1978) note, allowing asset sales to fully finance coupon interest payments makes analytical solutions intractable. The numerical method in this paper provides insights on equity, debt, and levered firm values when debt is permanent – that is, consol debt.

As Panel A of Figure 6 shows, equity values under total asset sales are higher than under proportional and no asset sales. When equity shareholders do not need to sell additional equity to finance coupon payments, default is only triggered when asset sales cannot meet the coupon payment. A very low default boundary implies that default is unlikely to happen and the firm can gain more tax benefits by increasing leverage or coupon interest payments without incurring large bankruptcy costs. Panels B and C of Figure 6 suggest that debt capacity and maximum levered firm value under the total asset sales are much higher than those under proportional and no asset sales.

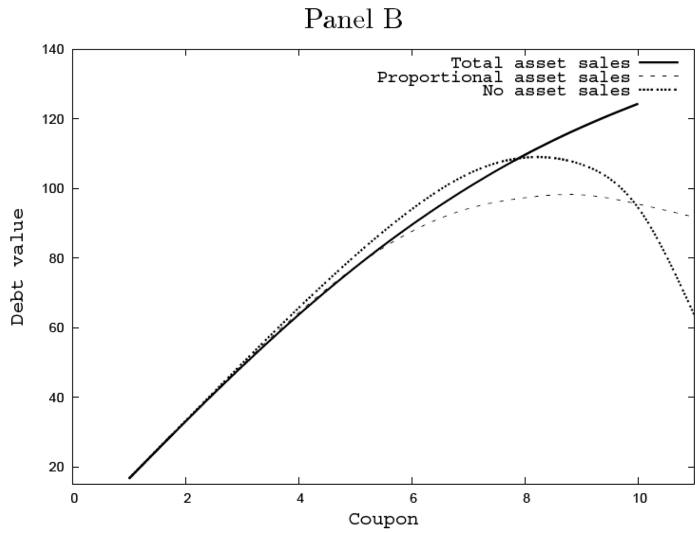
Figure 7 shows that asset sale restrictions are most likely that debt is short-term. When asset sales are used to fully service debt, levered firms need to sell assets to repay coupon interest and face value at debt maturity. A large increase in leverage or coupon interest payments will significantly increase the likelihood of default at maturity. Panels A and B of Figure 7 shows that the optimal leverage for finite maturity debt are much lower than optimal leverage under the consol debt case shown in Panels B and C of Figure 6.

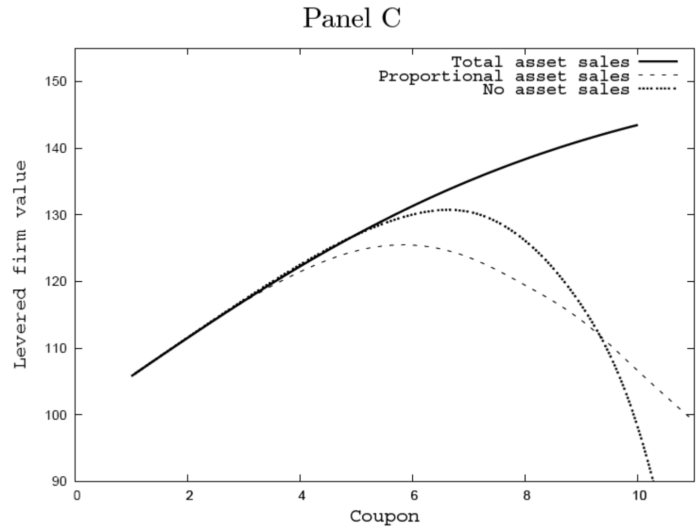
**Figure 6** Debt, equity, and levered firm values under asset sales and no asset sales




Notes: The solid, dashed, and dotted lines show debt, equity, and levered firm values under total, proportional, and no asset sales. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%. Debt is permanent.

**Figure 6** Debt, equity, and levered firm values under asset sales and no asset sales (continued)

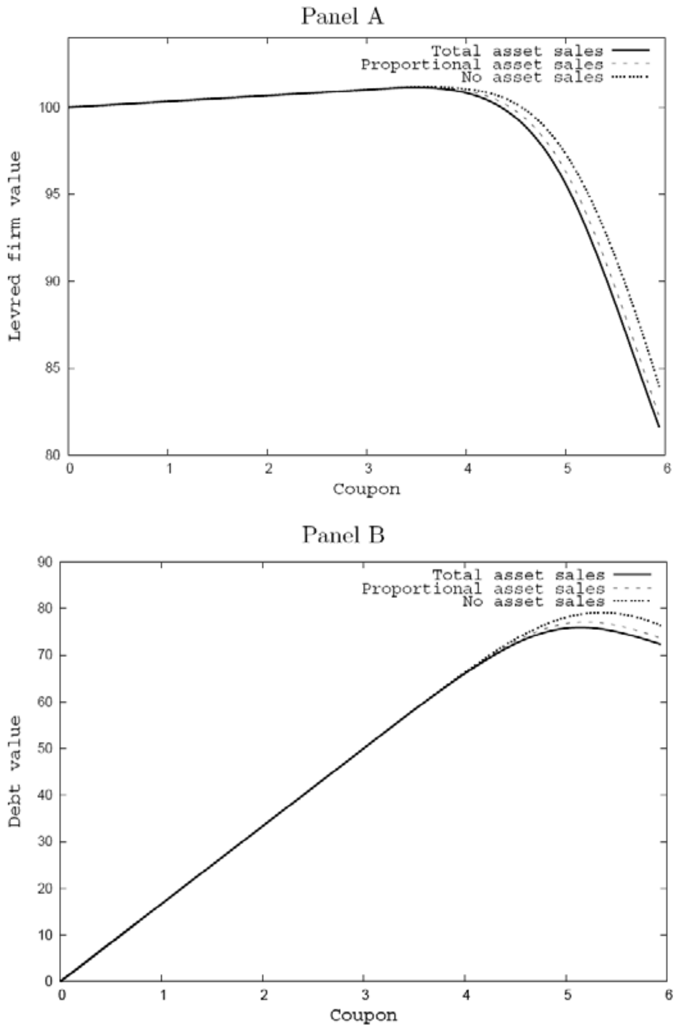



Notes: The solid, dashed, and dotted lines show debt, equity, and levered firm values under total, proportional, and no asset sales. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%. Debt is permanent.

For finite maturity debt, maximum levered firm value of \$101.014 and debt capacity of \$75.835 under total asset sales are lower than the maximum levered firm value of \$101.016 (\$101.062) and debt capacity of \$76.821 (\$78.109) under proportional asset sales (no asset sales). Asset sale restrictions in finite maturity bond covenants provide better protection for bondholders, and thereby, higher debt capacity.

**Figure 7** Debt and levered firm values under asset sales and no asset sales for finite maturity bonds




Notes: The solid, dashed, and dotted lines show debt, equity, and levered firm values under total, proportional, and no asset sales. The levered firm's asset value is initialised at \$100, the risk-free interest rate at 6%, asset volatility at 20%, bankruptcy cost at 50%, and corporate tax rate at 35%. Debt maturity is one year.

In Table 3, we analyse the influences of the situation that the tax refund is paid later than the corresponding coupon payment on both the equity and the debt values. The after-tax part of the coupon payment is financed by selling the firm's asset and the tax-benefit part is financed by issuing a short-term debt, say a commercial paper. This commercial paper is repaid when the firm receives the tax refund. The first row denotes the time span between the coupon payment date and the tax refund date (or the time span of the commercial paper). It can be observe that both the equity and debt values decrease with the increment of the time span of this commercial paper due to the increment of the interest expense paid to the commercial paper holder.

**Table 3** The impact of a later tax refund date (than the coupon payment date)

| C | *0 year* | | | *0.5 year* | | | *1 year* | | |
|---|---|---|---|---|---|---|---|---|---|
| | *Equity* | *Debt* | *Tax* | *Equity* | *Debt* | *Tax* | *Equity* | *Debt* | *Tax* |
| 7 | 34.941 | 100.057 | 35.014 | 34.222 | 99.580 | 33.817 | 33.535 | 99.108 | 32.661 |
| 8 | 28.889 | 109.354 | 38.266 | 28.183 | 108.702 | 36.912 | 27.511 | 108.077 | 35.615 |

Notes: The first column denotes the coupon interest payment *C*, and the first row denotes the time span between the coupon payment date and the tax refund date. The firm's asset value is initialised at \$100, the risk-free interest rate at 6%, the asset volatility at 20%, the bankruptcy cost at 50%, and the corporate tax rate at 35%.

## 5 Conclusions

This paper analyses how the retention of realised tax benefits and the manner in which retained tax benefits are invested impact: endogenous default boundaries; debt, equity, and levered firm values; optimal capital structure, when asset sales to finance coupon interest payments are allowed or restricted. Our numerical analysis shows that retaining realised tax benefits and investing them in risk-free assets result in higher debt capacity and maximum firm value. The positive net-worth bond covenant (or the protected debt) can prevent equity shareholders from increasing equity value by increasing firm risk at bondholders' expense. We also analyse how an asset sales option influences equity, debt, and levered firm values. For consol debt, maximum debt and levered firm value are higher when there are no restrictions on asset sales and positive net-worth bond covenants. But for finite maturity debt, restrictions on asset sales better protect bondholders and result in higher debt and levered firm values.

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## Notes

1 See Kraus and Litzenberger (1973).

2 Leland (1994) also considers the 'asset-sale' case which allows the firm to finance the coupon interest and dividend payments through asset sales. We will address this case in later discussion.

3 Similar arguments can be found in Brennan and Schwartz (1978).

4 This proportional asset-sale assumption is widely used (Kim et al., 1993; Hilberink and Rogers, 2002).

5 Under the assumption that the total coupon payment is completely financed through asset sales, the net cash outflow is the after-tax coupon interest payment; i.e.,

$$\begin{aligned} dV_A &= \left(\alpha V_A - C + \tau C\right)dt + \sigma V_A dz \\ &= \left(\alpha V_A - (1-\tau)C\right)dt + \sigma V_A dz. \end{aligned} \tag{4}$$

6 The only exception is $S = 0$ for the case when the coupon interest payment is fully financed through asset sales (total asset sales).

7 The only exception is that $V_A^+ = V_A^- - (1-\tau)C\Delta t$ when the coupon interest payment is fully financed through asset sales as previously noted in footnote 5.

8 The only exception is that $V_A^+ = V_A^- - C\Delta t$ when the coupon interest payment is fully financed through asset sales.

9 The formula for the endogenous default boundary in Leland (1994) also suggests that the default boundary decreases with the volatility of firm's asset value.

10 Leland (1994) also argues that the yields of junk bond may actually decline when firm riskiness increases.