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Accurate approximation formulas for stock options with discrete dividends

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Pricing options on a stock that pays discrete dividends has not been satisfactorily settled in the literature. Frishling (2002) shows that there are three different models to model stock price with discrete dividends, but only one of these models is close to reality and generates consistent option prices. We follow Frishling (2002) by calling this model Model 3. Unfortunately, there is no analytical option pricing formula for Model 3, and many popular numerical methods such as trees are inefficient when used to implement Model 3. A new stock price model is proposed in this article. To guarantee that the option prices generated by this new model are close to those generated by Model 3, the distributions of the new model at exdividend dates and maturity approximate the distributions of Model 3 at those dates. To achieve this, a discrete dividend in Model 3 is replaced by a continuous dividend yield that can be represented as a function of discrete dividends and stock returns in the new model. Thus, the new model follows a lognormal diffusion process and the analytical option pricing formulas can be easily derived. Numerical experiments show that our analytical pricing formulas provide accurate pricing results.

I. Introduction

Pricing options on dividend-paying stocks is a long-standing question. By assuming that the stock price follows a lognormal diffusion, Black and Scholes (1973) arrive at their groundbreaking option pricing model for nondividend-paying stocks. Merton (1973) extends the model to the case, where the underlying stock pays a nonstochastic continuous dividend yield. He defines the cost of carrying of a stock as the risk-free interest rate less the dividend yield, and the stock is assumed to grow at the cost of the carrying rate. This continuous dividend yield assumption is widely adopted for pricing options as in Krausz (1985), Barone-Adesi and Whaley (1987), Broadie and Detemple (1995, 1996), Shackleton and Wojakowski (2001), Chang and Shackleton (2003) and many others. However, almost all stock dividends are paid discretely rather than continuously. We call this dividend setting the discrete dividend if

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Applied Economics Letters ISSN 1350–4851 print/ISSN 1466–4291 online © 2007 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/13504850701604078

the amounts of future dividends are assumed to be known today. Pricing options on a stock that pays discrete dividends seems to be investigated first in Black (1975).

The discrete-dividend option pricing problem has drawn a lot of attention in the literature. Three popular models for this problem are discussed in Frishling (2002), and these three models are briefly introduced as follows.

Model 1 Roll (1977) suggests that the stock price is divided into two parts: the stock price minus the present value of future dividends over the life of the option and the present value of future dividends. The former part (call it net-of-dividend stock price) is assumed to follow a lognormal diffusion process, whereas the latter part is assumed to grow at the risk-free rate. Vanilla options can be computed by applying the Black–Scholes formula with the stock price replaced by the net-of-dividend stock price. Cox and Rubinstein (1985) also call this model the ad hoc adjustment.

Model 2 Musiela and Rutkowski (1997), following Heath and Jarrow (1988), suggest that the cum-dividend stock price, defined as the stock price plus the forward values of the dividends paid from today up to maturity, follows a lognormal diffusion process. Thus, vanilla options can be computed by applying the Black–Scholes formula by replacing the stock price with the cum-dividend stock price and by adding the forward values of the dividends prior to maturity to the strike price.

Model 3 The stock price jumps down with the amount of dividend paid at the exdividend date, and follows lognormal price process between two exdividend dates.

Although the above models address the discrete-dividend option pricing problem, Frishling (2002) shows that they are incompatible with each other and generate very different prices with the same inputs. A brief sketch is given to show why Model 1 always generates lower option prices than Model 3. Assume that the volatility input to both models is σ. Model 1 sets the volatility of the net-of-dividend stock price as σ, whereas Model 3 sets the volatility of the stock price as σ. The volatility of the stock price in Model 1 is lower than that in Model 3 as the volatility of the present value of future dividends, a component of the stock price, is assumed to be zero in Model 1. Model 1, therefore, produces lower option prices, and the price difference between these two models becomes larger as σ becomes larger. To remove this difference, Hull (2000) recommends that the volatility of the net-of-dividend stock price be adjusted by a simple formula. However, our article shows that the performance of Hull's volatility adjustment is mixed. Similarly, we can also infer that Model 2 produces higher option prices than Model 3 as Model 2 assigns the volatility of the forward values of the dividends (which is not a part of stock price) to be σ.

The first two models are widely accepted in the academic literature (Geske, 1979; Whaley, 1981, 1982; Carr, 1998; Chance *et al.*, 2002) partly because closed-form option pricing formulas can be easily derived. However, Frishling (2002) points out that only Model 3 can reflect the reality and provide more consistent option prices. His numerical results show that both Model 1 and Model 3 can produces unreasonable pricing results for American-style options and some exotic options. For example, he argues that Model 1 could incorrectly renders a down-and-out barrier option worthless simply because the net-of-dividend stock price reaches the barrier when the dividends are large enough. In reality, the option has a reasonable chance to survive since these dividends are paid later than today. On the other hand, although Model 3 is much closer to reality than the other two models, it does not allow closed-form solutions for European-style option prices. Model 3 can be implemented by some numerical methods such as the tree method. But, a naive application of the tree method results in a nonrecombining tree as in Fig. 1. Note that the tree size grows drastically with the number of exdividend dates. This unpleasant property renders the tree model inefficient.

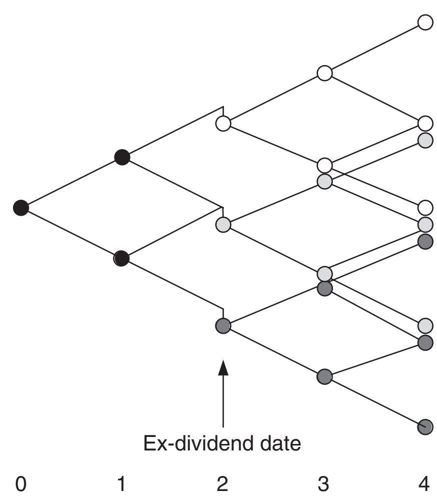

Fig. 1. A tree model for pricing stock options with discrete dividends
Notes: A discrete dividend is paid out at time step 2. There separate trees beginning at time step 2 are coloured in white, light gray and dark gray, respectively.

In additional to the first two models mentioned in Frishling (2002), efficient numerical algorithms and simple formulas can be constructed by approximating the discrete dividend with either (1) a fixed dividend yield on each exdividend date or (2) a fixed continuous dividend yield. Geske and Shastri (1985) construct a recombining tree by following the first approach. Although their tree model is efficient, numerical results in this article show that the pricing results can deviate significantly from the results of Model 3 in pricing European-style options. The second approach is followed by Chiras and Manaster (1978). They transform the discrete dividends into a fixed continuous dividend yield and then apply the Merton formula. As this approach can be shown to be equivalent to the first approach in pricing European-style options, it shares the same problem.

In this article, we will first construct a new stock price process (call it Model 4) that captures some important properties of Model 3. We then derive analytical pricing formulas for Model 4. To guarantee that the option prices generated by Model 4 are close to those generated by Model 3, the distributions of Model 4 at exdividend dates and maturity approximate the distributions of Model 3 at those dates. In fact, a discrete dividend paid at time τ in Model 3 is replaced by a proper continuous dividend yield paid from the last exdividend date (or option initial date) to time τ in Model 4. This continuous dividend yield is derived to be a function of discrete dividends and the stock returns by Taylor expansion to make the stock price (at exdividend date or at maturity) in Model 4 close to that in Model 3. The continuous dividend yield in Model 4 can be reinterpreted as the shift of the drift and the volatility of the stock return. Thus Model 4 follows the lognormal diffusion price process, and analytical option pricing formulas can be derived. Our approach can be easily extended to price an option with multiple discrete-dividend payouts. This property is useful as a stock can pay up to four dividends per annum in US, for example. Numerical results show that our pricing formulas can provide more accurate pricing results than other approximation methods mentioned above.

The article is organized as follows. The mathematical model is briefly covered in Section II. Model 4 and the corresponding pricing formulas are derived in Section III. We will first consider the single-discrete-dividend case and then extend our approach to the multiple-discrete-dividend case. Experimental results given in Section IV verify the accuracy of our pricing formulas. Section V concludes this article.

II. The Models

In Model 3, the stock price is assumed to follow the lognormal diffusion process in a risk-neutral economy:

$$\frac{\mathrm{d}S(t)}{S(t)} = r\mathrm{d}t + \sigma \mathrm{d}B(t)$$

where $S(t)$ denotes the stock price at time t, r denotes the annual risk-free interest rate, σ denotes the volatility, and $B(t)$ denotes the standard Brownian motion. Then the stock price $S(t)$ can be represented as

$$S(t) = S(s)e^{(r-0.5\sigma^2)(t-s)+\sigma(B(t)-B(s))}$$

if no dividend is paid between time s and time t. Assume that a discrete dividend D is paid at exdividend date τ. Then the stock price falls by the amount αD at time τ. For simplicity, α is assumed to be one in our pricing formulas. In general, α can be less than 1 when considering the effect of tax on dividend income. An $\alpha \neq 1$ poses no difficulties for modifying our pricing formulas.

Assume that a stock option initiates at time 0 and matures at time T. Then the payoff at time T is $(S(T)-X)^+$ for a vanilla call option and $(X-S(T))^+$ for a vanilla put option, where X denotes the strike price and $(A)^+$ denotes max$(A, 0)$. The underlying stock is assumed to pay n discrete dividends between time 0 and time T, where n is a positive integer. The i-th dividend c_i is paid at time $\sum_{j=1}^{i} t_j$, where t_j denotes the time span between the $(j-1)$-th exdividend date (for $j > 1$) or time 0 (for $j=1$) and the j-th exdividend date.

III. Analytical Formulas

We will first construct the stock price process for Model 4 in the single-discrete-dividend case and then derive an analytical pricing formula. For convenience, the stock price in Model 4 at time t is denoted as $S'(t)$. We further assume that $S'(0) \equiv S(0)$. Later, we will extend our work to the multiple-discrete-dividend case. Although our discussions focus on call options, extension to put options is straightforward.

A stock option with single discrete dividend

First, consider a stock that pays only one discrete dividend c_1 at time t_1 before maturity T. In Model 3, the stock price at time t_1 is

$S(t_1) = S(0)e^{\mu t_1+\sigma(B(t_1)-B(0))} - c_1$, where $\mu \equiv r - 0.5\sigma^2$. Thus the stock price $S(T)$ is expressed as

$$S(T) = \left[S(0)e^{\mu t_1+\sigma(B(t_1)-B(0))} - c_1\right]e^{\mu(T-t_1)+\sigma(B(T)-B(t_1))}$$

As $S(T)$ is no longer lognormally distributed, closed-form option pricing formulas become hard to come by.

The stock price process in Model 4 is designed to follow a lognormal price process. To achieve this, we first replace the discrete dividend c_1 paid at time t_1 by a properly chosen continuous dividend yield q_1 paid from time 0 to t_1 as follows:

$$S(t_1) = S(0)e^{\mu t_1+\sigma(B(t_1)-B(0))} - c_1 \equiv S(0)e^{(\mu-q_1)t_1+\sigma(B(t_1)-B(0))} \quad (1)$$

An approximation solution for q_1 is then derived to make

$$S(0)e^{(\mu-q_1)t_1+\sigma(B(t_1)-B(0))} \quad (2)$$

follow the lognormal distribution when we substitute this approximation solution into Equation 2. The approximation solution for q_1 is derived from Equation 1 as follows:

$$S(0)e^{\mu t_1+\sigma(B(t_1)-B(0))}(1 - e^{-q_1 t_1}) = c_1$$

$$\Rightarrow 1 - e^{-q_1 t_1} - \frac{c_1 e^{-\mu t_1}}{S(0)} e^{-\sigma(B(t_1)-B(0))}$$

The left-hand side of the above equation can be approximated by the first-order Taylor expansion as $1-(1-q_1t_1)$, and the right-hand side is approximated by $k_1(1-\sigma(B(t_1)-B(0)))$, where $k_1 = c_1 e^{\mu t_1}/S(0)$. Thus we have $q_1 \approx k_1(1-\sigma(B(t_1)-B(0)))/t_1$.

Finally, $S(T)$ can be approximated by $S'(T)$ as follows:

$$\begin{aligned} S(T) &= [S(0)e^{\mu t_1+\sigma(B(t_1)-B(0))} - c_1]e^{\mu(T-t_1)+\sigma(B(T)-B(t_1))} \\ &= S(0)e^{(\mu-q_1)t_1+\sigma(B(t_1)-B(0))+\mu(T-t_1)+\sigma(B(T)-B(t_1))} \\ &\approx S'(0)e^{(\mu-k_1/T)T+k_1\sigma(B(t_1)-B(0))+\sigma(B(T)-B(0))} \equiv S'(T) \end{aligned}$$

Note that $S'(T)$ follows the lognormal distribution. Let $\mathrm{Var}(X)$ denote the variance of the random variable X. Define σ_1 by

$$\begin{aligned} \sigma_1 &\equiv \sqrt{\frac{Var[k_1\sigma(B(t_1)-B(0)) + \sigma(B(T)-B(0))]}{T}} \\ &= \sqrt{\frac{(1+k_1)^2\sigma^2 t_1 + \sigma^2(T-t_1)}{T}}. \end{aligned}$$

Then the discrete dividend c_1 paid at time t_1 in Model 3 is replaced by a continuous dividend yield q_1 that can be approximately interpreted as the shift of the drift of the stock return from μ to $\mu - k_1/T$ and the volatility from σ to σ_1 in Model 4. The value for a vanilla call option can be calculated by the risk-neutral variation method as follows:

$$e^{-rT}E(S'(T) - X)^+ \quad (3)$$

$$\begin{aligned} &= e^{-rT}E[S'(0)e^{(\mu-k_1/T)T+k_1\sigma(B(t_1)-B(0))+\sigma(B(T)-B(0))} - X]^+ \\ &= S'(0)e^a N(d_1) - Xe^{-rT}N(d_2) \quad (4) \end{aligned}$$

where $a = \sigma_1^2 - \sigma^2/2T - k_1$, $d_1 = \ln S'(0)/X - k_1 + (\mu+\sigma_1^2)T/\sigma_1\sqrt{T}$, and $d_2 = d_1 - \sigma_1\sqrt{T}$.

The accuracy of above formula can be improved by expanding $k_1 e^{-\sigma(Bt_1)-B(0))}$ further as $k_1(1-\sigma[B(t_1)- B(0)] + \sigma^2[B(t_1)-B(0)]^2/2)$. To make Model 4 follow a lognormal price process, q_1 is derived as follows:

$$\begin{aligned} q_1 &\approx \frac{k_1\left[1-\sigma(B(t_1)-B(0)) + \sigma^2\frac{B(t_1)-B(0))^2}{2}\right]}{t_1} \\ &\approx \frac{k_1[1-\sigma(B(t_1)-B(0))] + \delta_1}{t_1} \end{aligned} \quad (5)$$

where $\delta_1 \equiv E(k_1\sigma^2[B(t_1)-B(0)]^2/2) = k_1\sigma^2 t_1/2$. Thus, $S'(T)$ can be derived as follows:

$$S'(T) \equiv S'(0)e^{(\mu-k_1/T)T+\sigma(B(T)-B(0))+k_1\sigma(B(t_1)-B(0))-\delta_1} \quad (6)$$

A more accurate formula for a call option is then obtained by substituting Equation 6 into Equation 3. The resulting pricing formula can be expressed in terms of Equation 4 with a and d_1 redefined as $\sigma_1^2 - \sigma^2/2T - k_1 - \delta_1$, and $\ln S(0)/X - k_1 + (\mu+\sigma_1^2)T - \delta_1/\sigma_1\sqrt{T}$, respectively. Numerical experiments in Section IV show that this formula generates accurate prices.

Multiple discrete dividends

The aforementioned approach can be further extended to price a stock option with multiple discrete dividends. We will first consider the two-discrete-dividend case and then describe the generalized pricing formula for the multiple-discrete-dividend case without proof.

Assume that two discrete dividends c_1 (paid at time t_1) and c_2 (paid at time t_1+t_2) are paid prior to time T. We again replace the dividend c_2 paid at time t_1+t_2 by a proper continuous dividend yield q_2 paid from time t_1 to time t_1+t_2 as follows:

$$
\begin{aligned}
&S(t_1)e^{\mu t_2+\sigma(B(t_1+t_2)-B(t_1))}-c_2\\
&\quad\equiv S(t_1)e^{(\mu-q_2)t_2+\sigma(B(t_1+t_2)-B(t_1))}\\
&\quad\Rightarrow 1-e^{-q_2t_2}=\frac{c_2e^{-\mu t_2}}{S(t_1)}e^{-\sigma(B(t_1+t_2)-B(t_1))}
\end{aligned}
\tag{7}
$$

By substituting Equation 1 into Equation 7 with $q_1\approx k_1[1-\sigma(B(t_1)-B(0))]+\delta_1/t_1$, q_2 can be derived as follows:

$$
q_1\approx\frac{k_2[1-(1+k_1)\sigma(B(t_1)-B(0))-\sigma(B(t_2)-B(t_1))]+\delta_2}{t_2}
\tag{8}
$$

where $k_2\equiv c_2e^{-\mu(t_1+t_2)+k_1+\delta_1}/S(0)$ and $\delta_2\equiv k_2[(1+k_1)^2\sigma^2t_1+\sigma^2t_2]/2$. Thus $S(T)$ can be approximated by $S'(T)$ as follows:

$$
\begin{aligned}
S(T)&=\left(S(t_1)e^{\mu t_2+\sigma(B(t_1+t_2)-B(t_1))}-c_2\right)\\
&\quad\times e^{\mu(T-t_1-t_2)+\sigma(B(T)-B(t_1+t_2))}\\
&=S(t_1)e^{(\mu-q_2)t_2+\sigma(B(t_1+t_2)-B(t_1))}\\
&\quad\times e^{\mu(T-t_1-t_2)+\sigma(B(T)-B(t_1+t_2))}\\
&=S(0)e^{(\mu-q_1)t_1+\sigma(B(t_1)-B(0))}e^{(\mu-q_2)t_2+\sigma(B(t_1+t_2)-B(t_1))}\\
&\quad\times e^{\mu(T-t_1-t_2)+\sigma(B(T)-B(t_1+t_2))}\\
&\approx S'(0)e^{\left(\mu-\frac{k_1+k_2+\delta_1+\delta_2}{T}\right)T+(1+k_1+k_2+k_1k_2)\sigma(B(t_1)-B(0))+(1+k_2)\sigma(B(t_1+t_2)-B(t_1))+\sigma(B(T)-B(t_1+t_2))}\\
&\equiv S'(T)
\end{aligned}
\tag{9}
$$

where we substitute Equations 5 and 8 into Equation 9. Note that $S'(T)$ follows the lognormal distribution. Define σ_2 by

$$
\begin{aligned}
\sigma_2&\equiv\sqrt{\frac{Var[(1+k_1+k_2+k_1k_2)\sigma(B(t_1)-B(0))+(1+k_2)\sigma(B(t_1+t_2)-B(t_1))+\sigma(B(T)-B(t_1+t_2))]}{T}}\\
&=\sqrt{\frac{(1+k_1+k_2+k_1k_2)^2\sigma^2t_1+(1+k_2)^2\sigma^2t_2+\sigma^2(T-t_1-t_2)}{T}}
\end{aligned}
$$

Again, the discrete dividends c_1 and c_2 in Model 3 are replaced by continuous dividend yields q_1 and q_2 that can be approximately interpreted as the shift of the drift of the stock return from μ to $\mu-k_1+k_2+\delta_1+\delta_2/T$ and the volatility from σ to σ_2 in Model 4. The value for a vanilla call option can be calculated by the risk-neutral variation method as follows:

$$
\begin{aligned}
e^{-rT}E(S'(T)-X)^+=S'(0)e^{(\sigma_2^2-\sigma^2/2)T-k_1-k_2-\delta_1-\delta_2}N(d_1')\\
-Xe^{-rT}N(d_2')
\end{aligned}
$$

where $d_1'=\ln S'(0)/X-k_1-k_2+(\mu+\sigma_2^2)T-\delta_1-\delta_2/\sigma_2\sqrt{T}$ and $d_2'=d_1'=\sigma_2\sqrt{T}$.

The pricing formula for stock options with more discrete dividends can be derived by iteratively repeating the aforementioned steps. The pricing formula for n-discrete-dividend case is illustrated with the aid of recursive formulas below:

$$
\begin{aligned}
a_{i,j}&=\begin{cases}0, & \text{if } i>j,\\ \sigma & \text{if } i=j,\\ \sum_{h=1}^{j-1}a_{i,h}k_h+\sigma & \text{if } i<j,\end{cases}\\
\delta_i&=\frac{k_i\sum_{j=1}^{i}a_{j,i}^2t_j}{2},\\
k_i&=\frac{c_ie^{-\mu\sum_{j=1}^{i}t_i+\sum_{j=1}^{i-1}(k_j+\delta_j)}}{S'(0)}\\
\sigma_i&=\sqrt{\frac{\sum_{j=1}^{i}a_{j,i+1}^2t_j+a_{i+1,i+1}^2(T-\sum_{h=1}^{i}t_h)}{T}}.
\end{aligned}
$$

The pricing formula of a vanilla call option is then

$$
S'(0)e^{\frac{\sigma_n^2-\sigma^2}{2}T-\sum_{i=1}^{n}(k_i+\delta_i)}\mathrm{N}(d_1'')-Xe^{-rT}\mathrm{N}(d_2''),
$$

where $d_1''=\ln S'(0)/X+(\mu+\sigma_n^2)T-\sum_{i=1}^{n}(k_i+\delta_1)/\sigma_n\sqrt{T}$ and $d_2''=d_1''-\sigma_n\sqrt{T}$.

IV. Numerical Results

We compare Geske and Shastri's fixed dividend yield model, Hull's volatility adjustment model, and all the four discrete dividend models mentioned earlier in

Table 1. Pricing a call option with single discrete dividend

	0.4						0.5					
X	FDY	Model1	Hull	Model2	Model4	Model3	FDY	Model1	Hull	Model2	Model4	Model3
95	*16.263	*16.336	17.090	17.112	16.875	16.933	*19.890	*19.969	20.901	20.937	20.643	20.843
100	*14.214	*14.270	15.044	15.048	14.815	14.754	*17.964	*18.003	*18.959	*18.971	18.687	18.584
105	*12.400	*12.439	*13.222	*13.206	12.982	12.989	*16.194	*16.222	17.194	17.182	16.910	16.929

Notes: The initial stock price is 100, the risk-free rate is 3%, the time to maturity is 1 year, and a five-dollar-dividend is paid at year 0.6.The volatilities of the stock price are shown in the first row. The strike prices are listed in the first column.FDY denotes the fixed dividend yield approach of Geske and Shastri (1985). Model1, ..., Model4 denote the option prices generated by Model 1, ..., Model 4, respectively.Hull denotes volatility adjustment approach of Hull (2000).Option prices that deviate from Model 3 by 0.3 are marked by asterisks.

Table 2. Pricing a call option with two discrete dividends

	0.4						0.5					
X	FDY	Model1	Hull	Model2	Model4	Model3	FDY	Model1	Hull	Model2	Model4	Model3
95	*16.303	*16.336	17.090	17.112	16.849	16.836	*19.931	*19.969	*20.901	*20.937	20.620	20.549
100	*14.250	*14.270	*15.044	*15.048	14.792	14.733	*18.001	*18.003	*18.959	*18.971	18.667	18.621
105	*12.433	*12.439	*13.222	*13.206	12.963	12.883	*16.228	*16.222	*17.194	*17.182	16.829	16.829

Notes: The numerical settings are the same as those settings in Table 1 except that a 2.5-dollar-dividend is paid at year 0.4 and year 0.8.Option prices that deviate from Model3 by 0.3 are marked by asterisks.

this article. Geske and Shastri (1985) use fixed dividend yields to approximate discrete dividends. The fixed dividend yield is defined as the discrete dividend amount divided by the initial stock price. For example, the dividend yield is 5% if the initial stock price is 100 and the discrete dividend is 5. We use FDY to denote their approach. Model 1 generates lower option prices than Model 3 as argued before. To remove this difference, Hull (2000) recommends that the volatility of net-of-dividend stock price be adjusted by the volatility of the stock price multiplied by $S(0)/(S(0)-D)$, where D denotes the present value of future dividends paid between time 0 to time T. We use Hull to denote Hull's volatility adjustment approach. Besides, we use Model1, ..., Model4 to denote the option prices generated by Model 1, ..., Model 4. The option prices generated by Model 3 are produced by the Monte Carlo simulation based on 100 000 trials.

The numerical results for these models are listed in Table 1 and 2, where Table 1 focuses on the single-discrete-dividend case and Table 2 focuses on the two-discrete-dividend case. All the prices that deviate from Model3 by 0.3 are marked by asterisks. It is not surprising that the option prices generated by Model 2 are higher than the prices generated by Model 3. On the other hand, Model 1 generates lower option prices than Model 3. The difference among these three models becomes larger as the volatility increases. FDY does not approximate Model 3 well as it produces lower option prices than Model 1. Hull's volatility adjustment approach seems to overprice the options. It can be observed that only Model 4 produces options prices that are close to those generated by Model 3.

The option price generated by Model 3 in each case in Table 2 is lower than that in the corresponding case (except one case) in Table 1. Model 4 successfully captures this trend, but all other models fail. Note that both Model 1 and Hull's volatility adjustment approach produce similar option prices in the single-discrete-dividend case and the two-discrete-dividend case. This is because the net-of-dividend stock price in the single-discrete-dividend case ($= 100 - 5e^{-0.03\times 0.6}$) is almost equal to that in the multiple-discrete-dividend case ($= 100 - 2.5e^{-0.03\times 0.4} - 2.5e^{-0.03\times 0.8}$). Model 2 also produces similar option prices in both cases since the cum-dividend stock prices for both cases are almost equal.

V. Conclusions

Traditional models for pricing options on discrete-dividend-paying stocks either produce inconsistent pricing results or are inefficient. Our article constructs a new stock price model by replacing discrete dividends with proper continuous dividend yields which can be viewed as functions of discrete dividends and stock returns. This model follows a lognormal diffusion process and analytical pricing formulas can be easily derived. Numerical results verify the superiority of our approach.

Acknowledgements

We thank Ren-Her Wang for useful suggestions.

The author was supported in part by NSC grant 94-2213-E-033-024 and NCTU research grant for financial engineering and risk management project.

The author was supported in part by NSC grant 95-2213-E-002-044.

References

Barone-Adesi, G. and Whaley, R. E. (1987) Efficient analytic approximation of american options values, *Journal of Finance*, **42**, 301–20.

Black, F. (1975) Fact and fantasy in the use of options, *Financial Analysts Journal*, **31**, 61–72.

Black, F. and Scholes, M. (1973) The pricing of options and corporate liabilities, *Journal of Political Economy*, **81**, 637–59.

Broadie, M. and Detemple, J. B. (1995) American capped call options on dividend paying assets, *Review of Financial Studies*, **8**, 161–91.

Broadie, M. and Detemple, J. B. (1996) American options valuation: new bounds. approximations and a comparison of existing methods, *Review of Financial Studies*, **9**, 1211–50.

Carr, P. (1998) Randomization and the American put, *The Review of Financial Studies*, **11**, 597–626.

Chance, D.M., Kumar, R. and Rich, D. (2002) European option pricing with discrete stochastic dividends, *Journal of Derivatives*, **9**, 39–45.

Chang, S.-L. and Shackleton, M. (2003) The simplest american and real option approximations: Geske–Johnson interpolation in maturity and yield, *Applied Economics Letters*, **10**, 709–16.

Chiras, D. P. and Manaster, S. (1978) The informational content of option prices and a test of market efficiency, *Journal of Financial Economics*, **6**, 213–34.

Cox, J. C. and Rubinstein, M. (1985) *Options Markets*, NJ: Prentice-Hall, Englewood Cliffs.

Frishling, V. (2002) A discrete question, *Risk*, [redacted], 115–6. 3

Geske, R. (1979) A note on an analytical valuation formula for unprotected american call options on stocks with known dividends, *Journal of Financial Economics*, **7**, 375–80.

Geske, R. and Shastri, K. (1985) Valuation by approximation: a comparison of alternative option valuation techniques, *Journal of Financial and Quantitative Analysis*, **20**, 45–71.

Heath, D. and Jarrow, R. (1988) Exdividend stock price behaviour and arbitrage opportunities, *Journal of Business*, **61**, 95–108.

Hull, J. (2000) *Options, Futures, and Other Derivatives,* , 4th edn, NJ: Prentice-Hall, Englewood Cliffs.

Krausz, J. (1985) Option parameter analysis and market efficiency tests: a simultaneous solution approach, *Applied Economics*, **17**, 885–96.

Merton, R. C. (1973) Theory of rational option pricing, *Bell Journal of Economics and Management Science*, **4**, 141–83.

Musiela, M. and Rutkowski, M. (1997) *Martingale Methods in Financial Modeling*, Springer, Germany.

Roll, R. (1977) An analytic valuation formula for unprotected american call options on stocks with known dividends, *Journal of Financial Economics*, **5**, 251–8.

Shackleton, M. and Wojakowski, R. (2001) On the expected payoff and true probability of exercise of european options, *Applied Economics Letters*, **8**, 269–71.

Whaley, R. E. (1981) On the valuation of american call options on stocks with known dividends, *Journal of Financial Economics*, **9**, 207–11. 1

Whaley, R. E. (1982) Valuation of american call options on dividend-paying stocks: empirical tests, *Journal of Financial Economics*, **10**, 29–58. 1