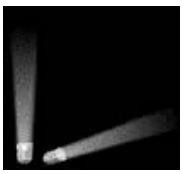
Selection Trees



Winner trees.

Loser Trees.

Winner Trees

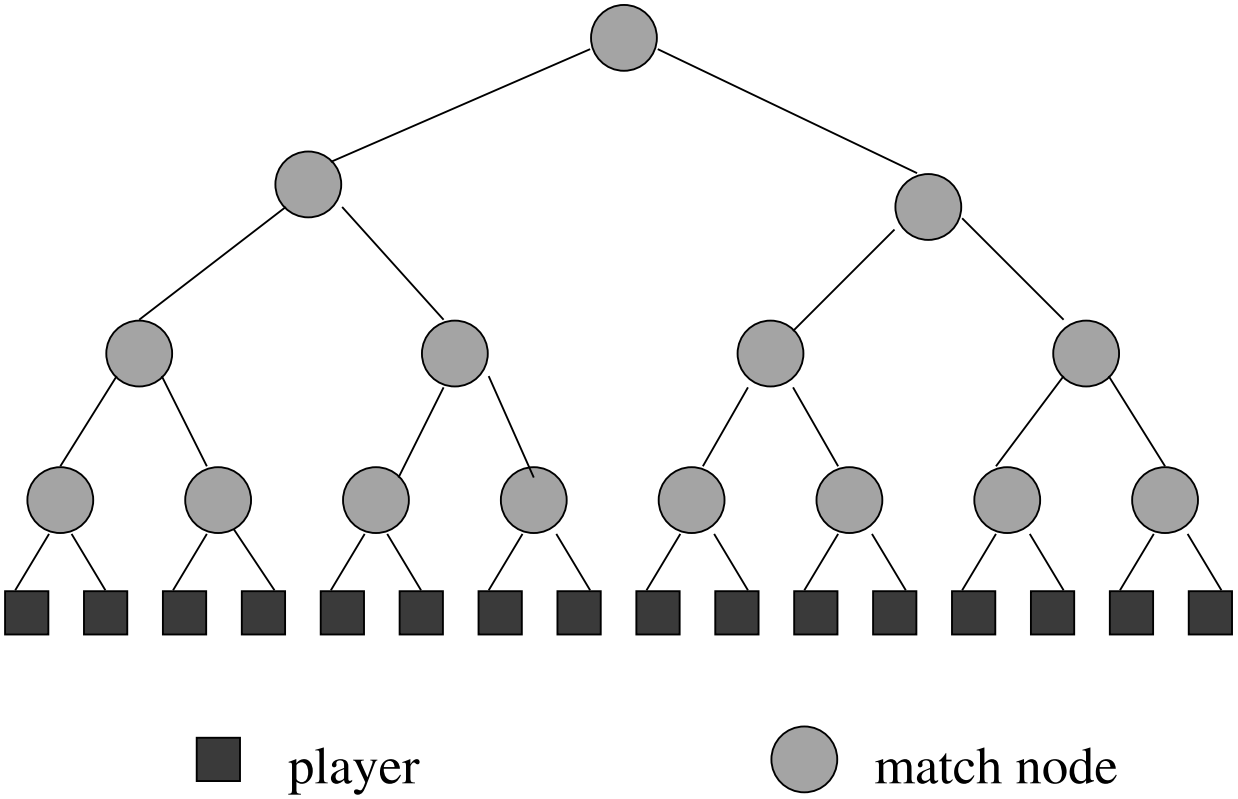
Complete binary tree with k external nodes and k - 1 internal nodes.

External nodes represent tournament players.

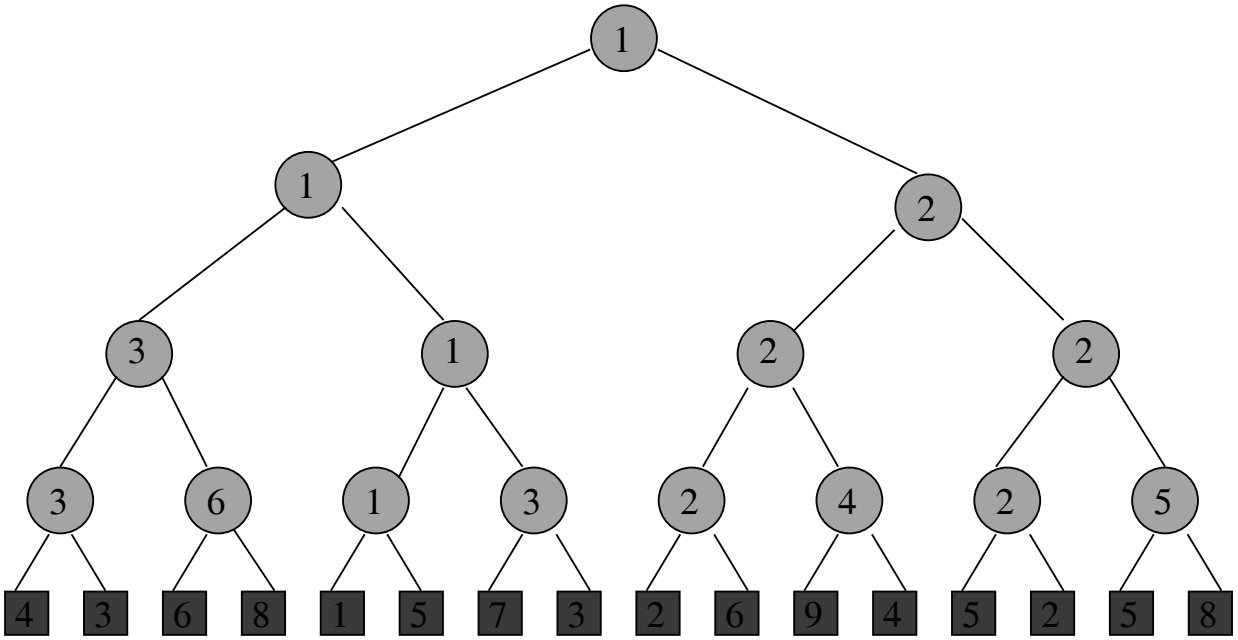
Each internal node represents a match played between its two children; the winner of the match is stored at the internal node.

Root has overall winner.

Winner Tree For 16 Players

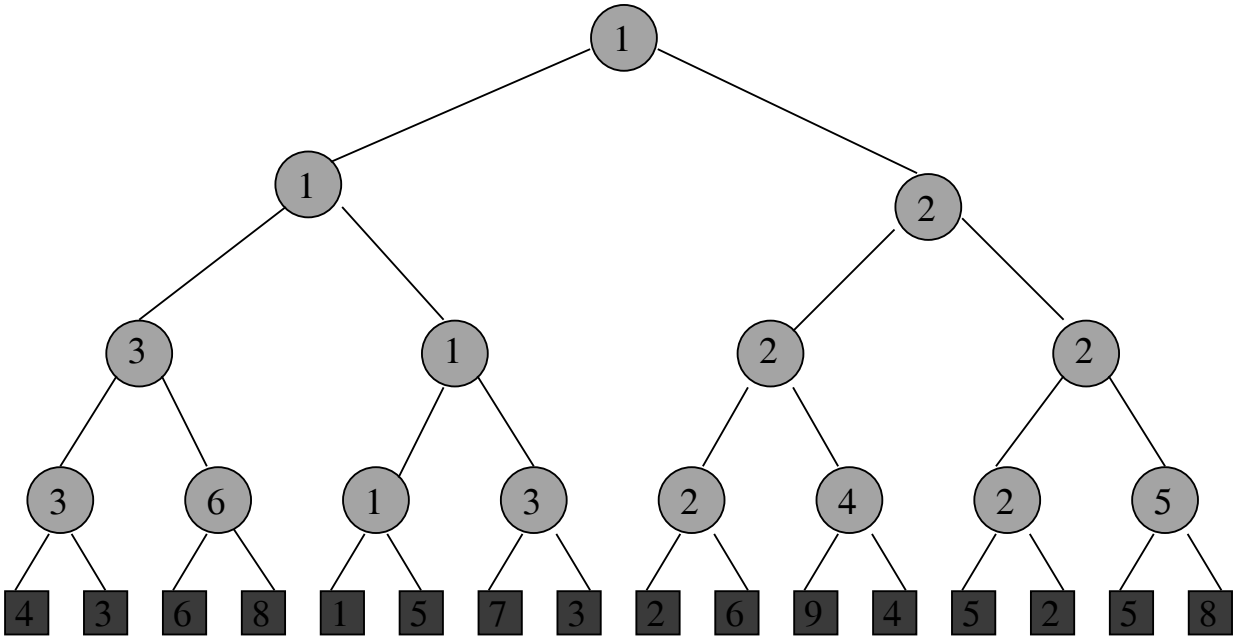


Winner Tree For 16 Players



Smaller element wins => min winner tree.

Winner Tree For 16 Players



height is $\log_2 n$ (excludes player level)

Complexity Of Initialize

- O(1) time to play match at each match node.
- k - 1 match nodes.
- O(k) time to initialize k player winner tree.

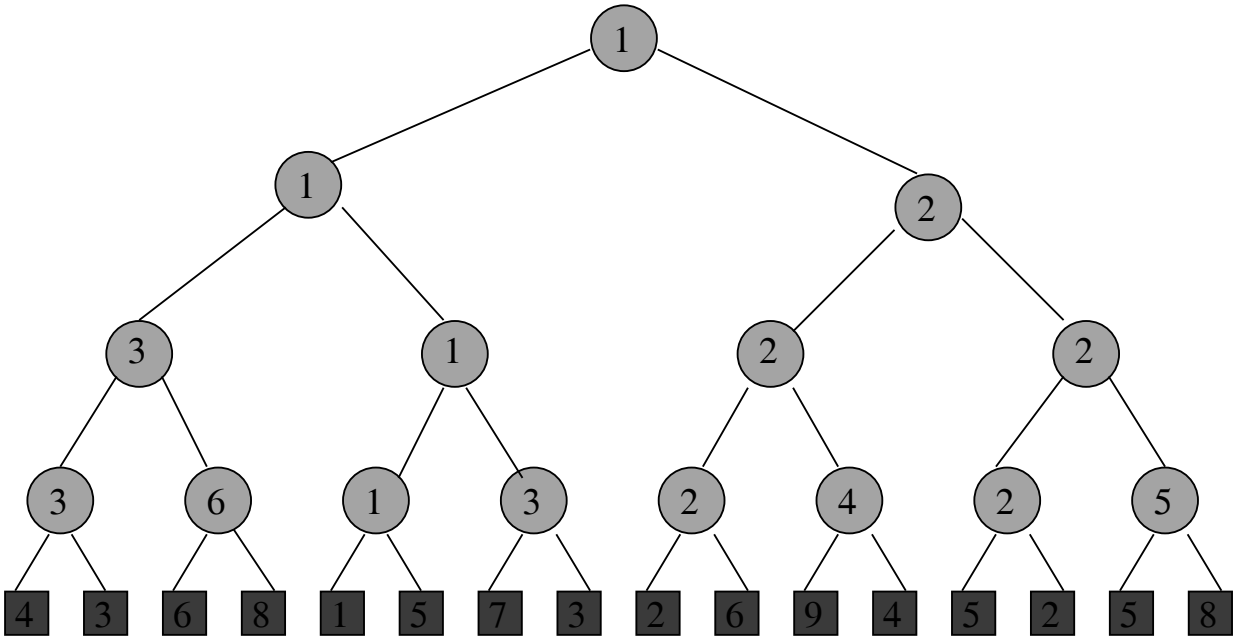
Applications

Sorting.

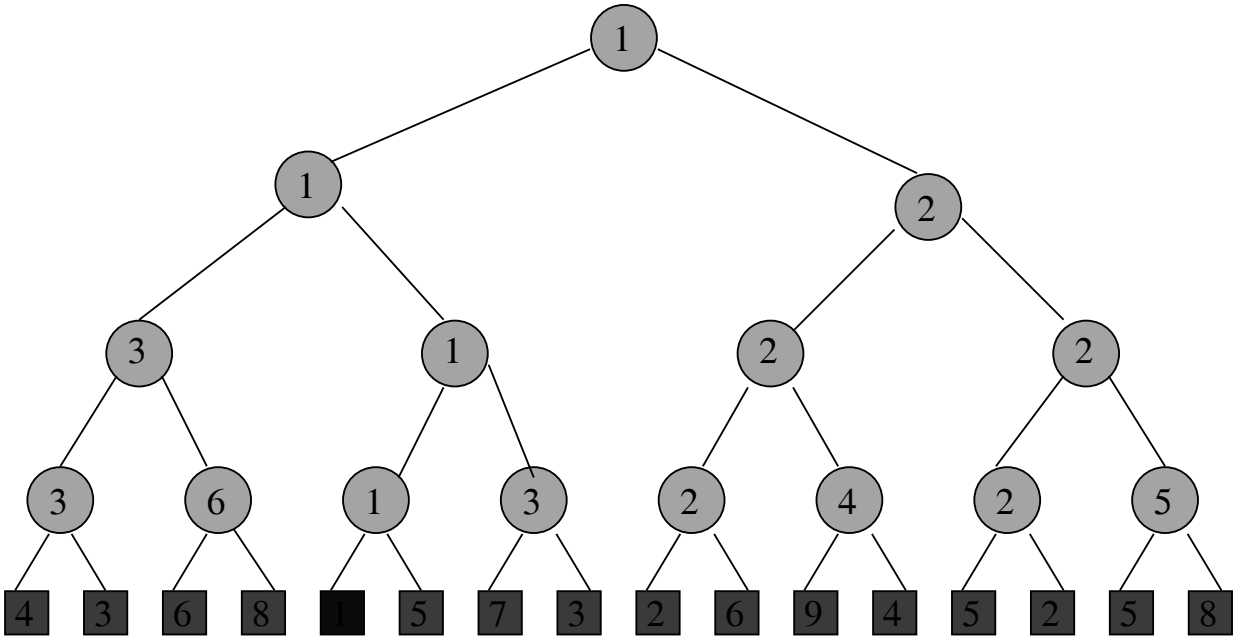
Insert elements to be sorted into a winner tree.

Repeatedly extract the winner and replace by a large value.

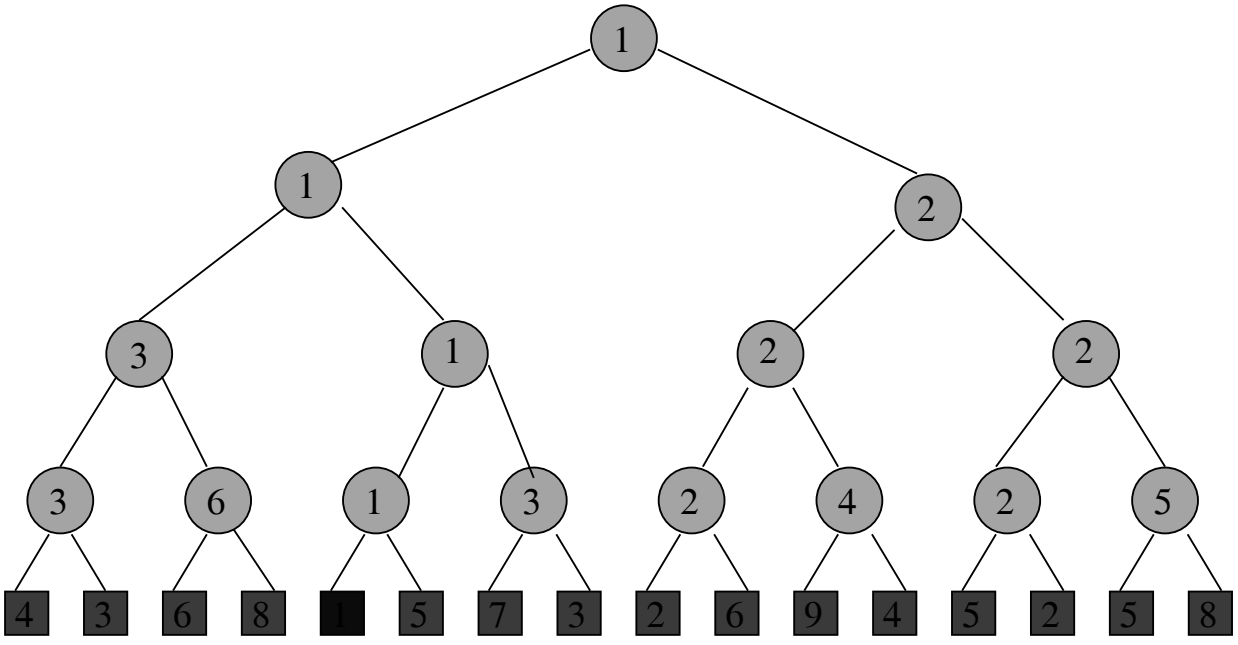
Sort 16 Numbers



Sort 16 Numbers

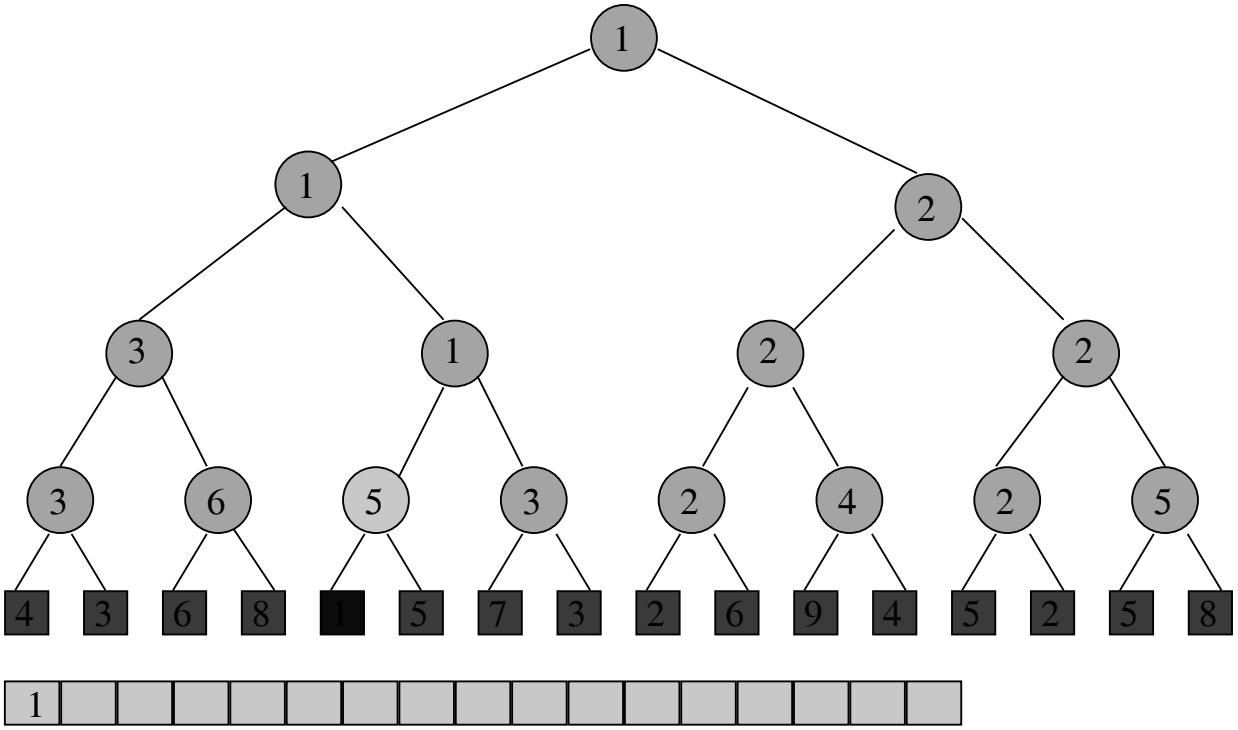


Sort 16 Numbers



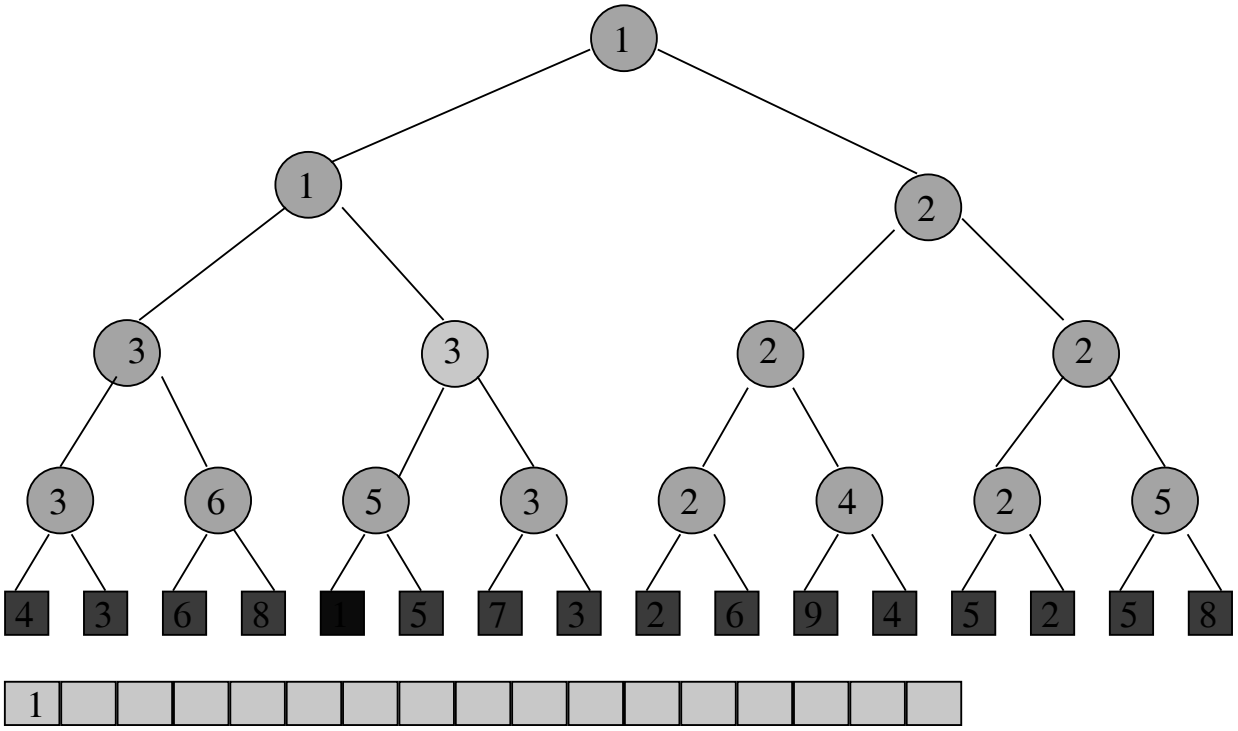
Sorted array.

Sort 16 Numbers



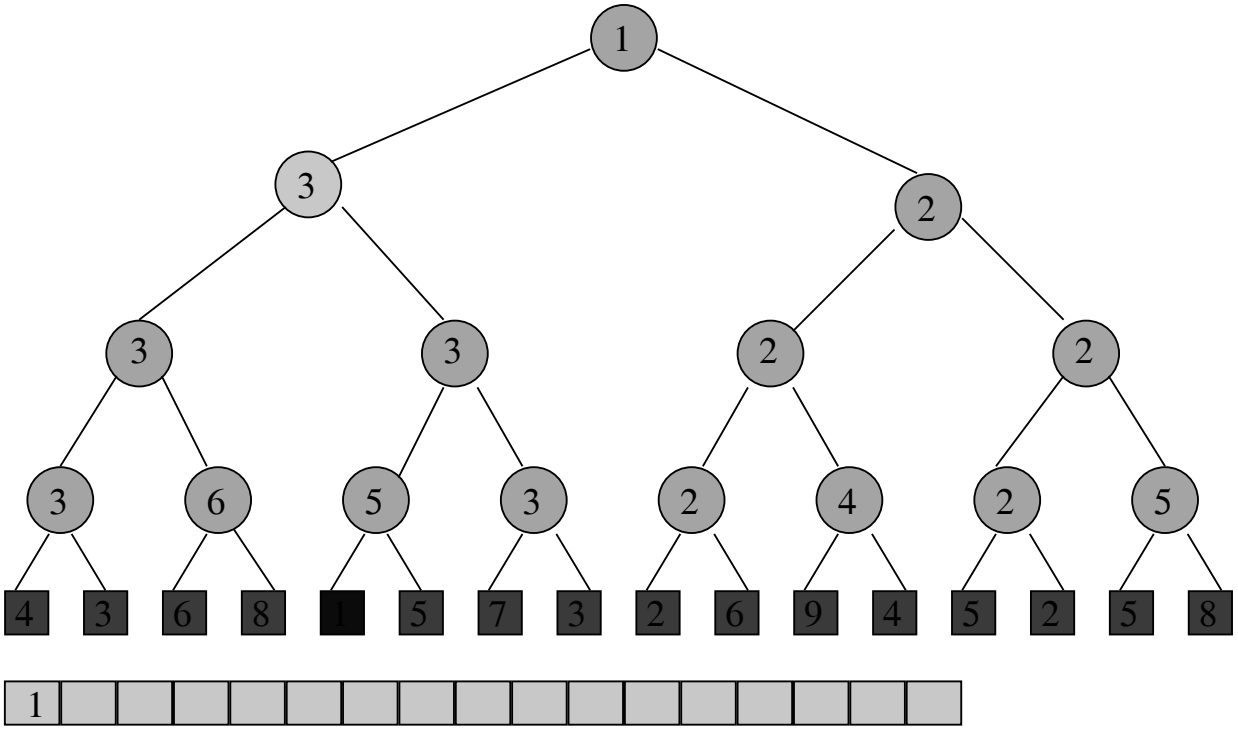
Sorted array.

Sort 16 Numbers



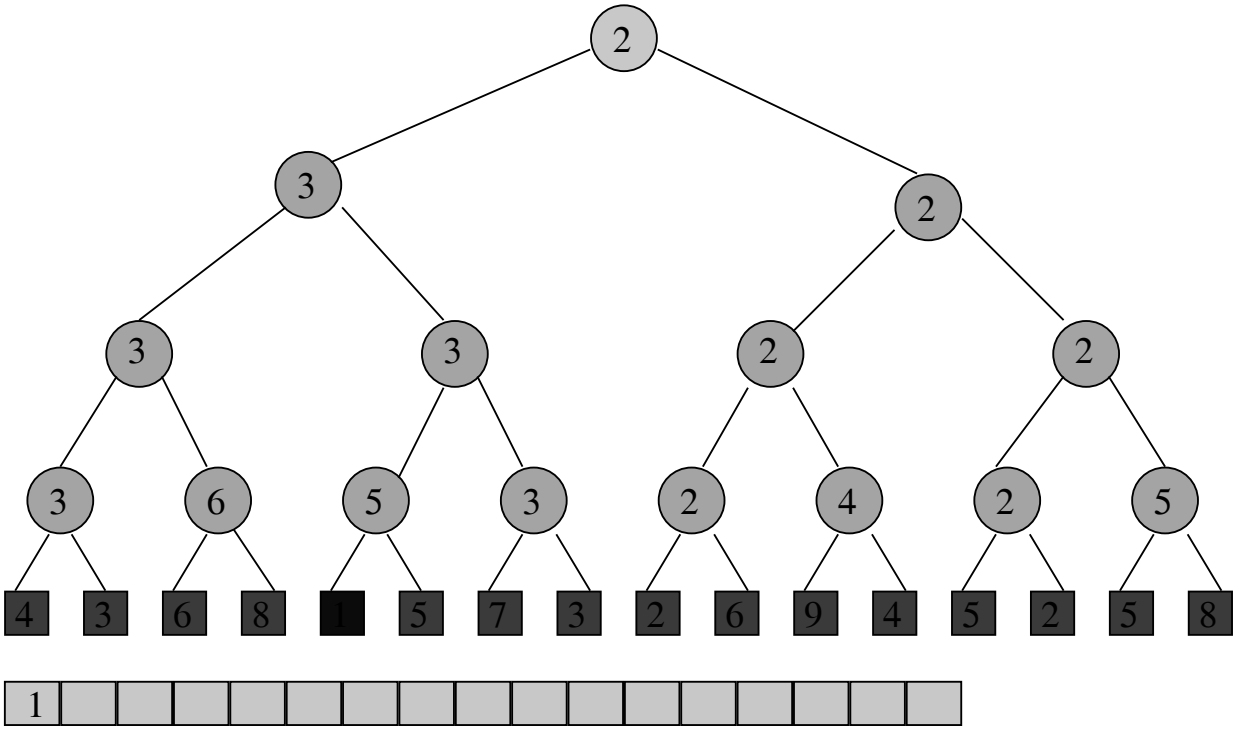
Sorted array.

Sort 16 Numbers



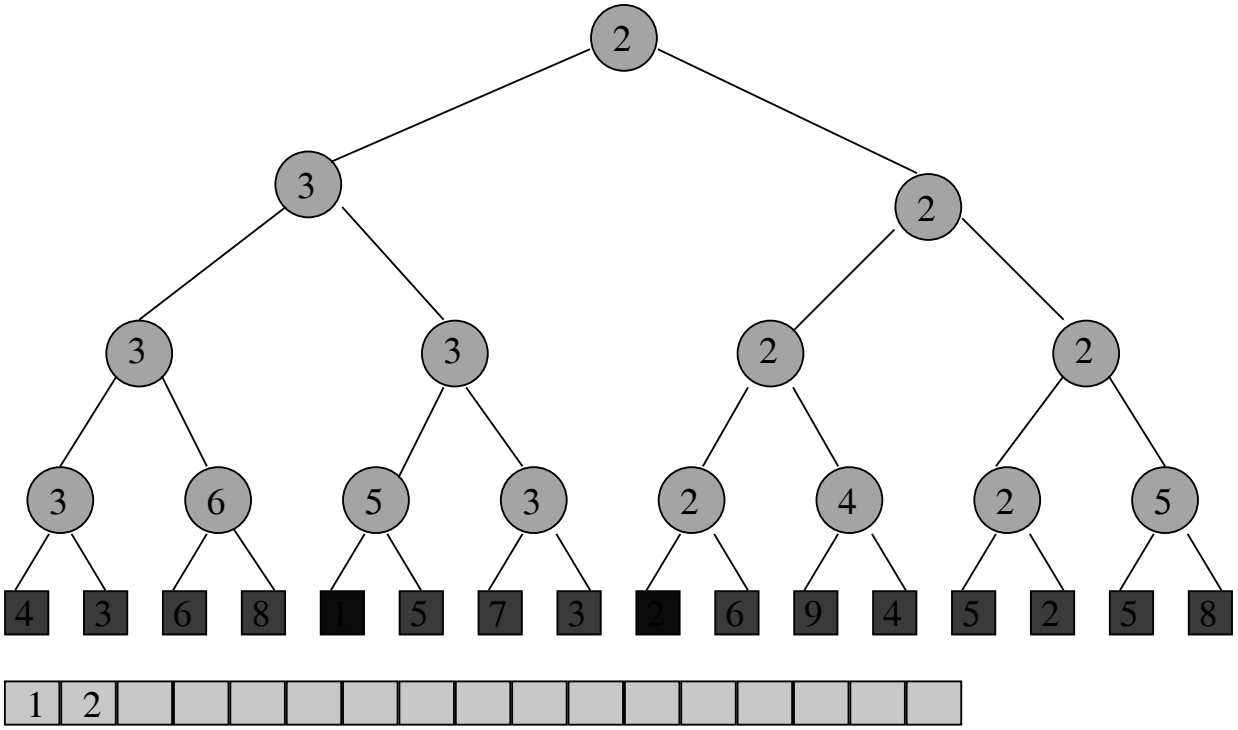
Sorted array.

Sort 16 Numbers



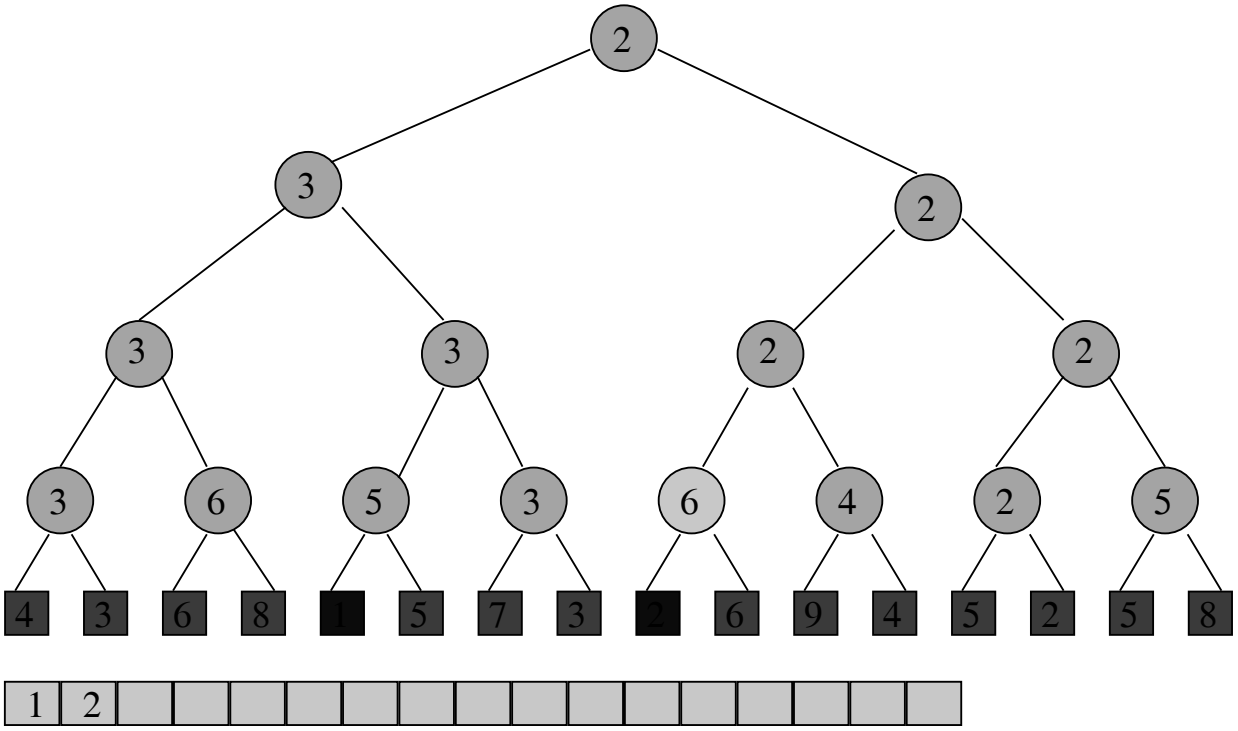
Sorted array.

Sort 16 Numbers



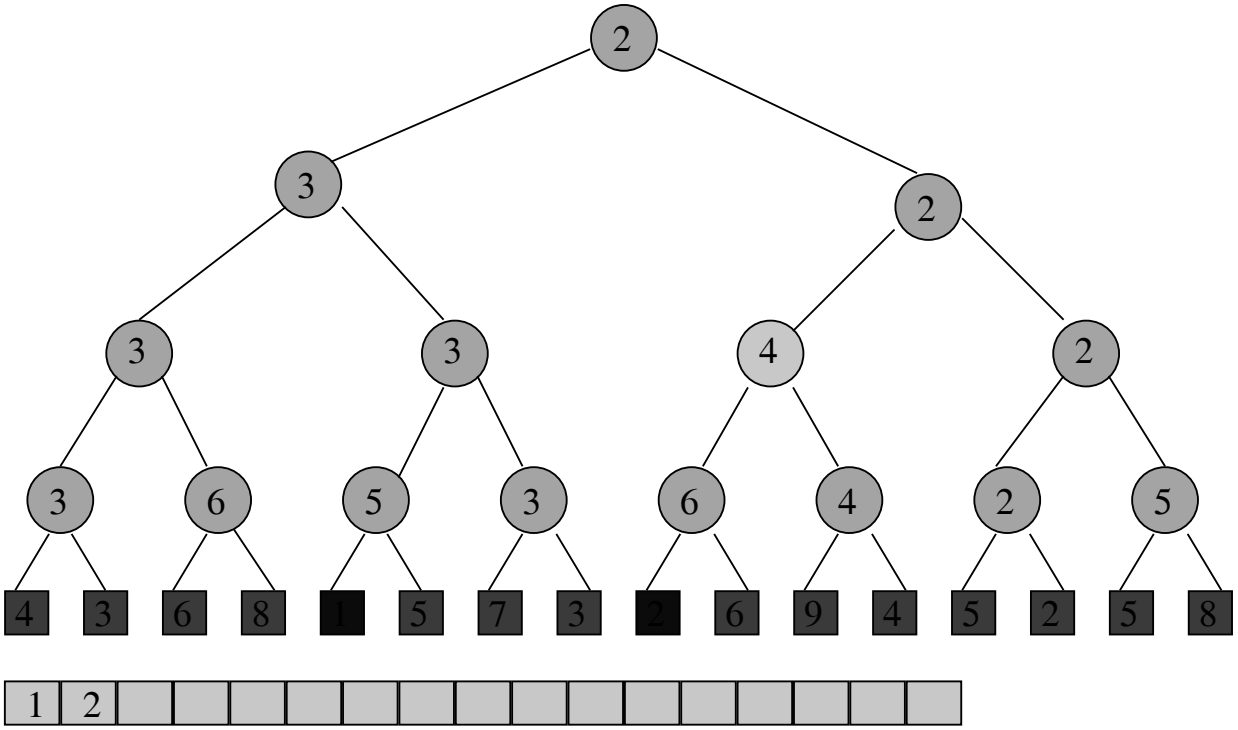
Sorted array.

Sort 16 Numbers



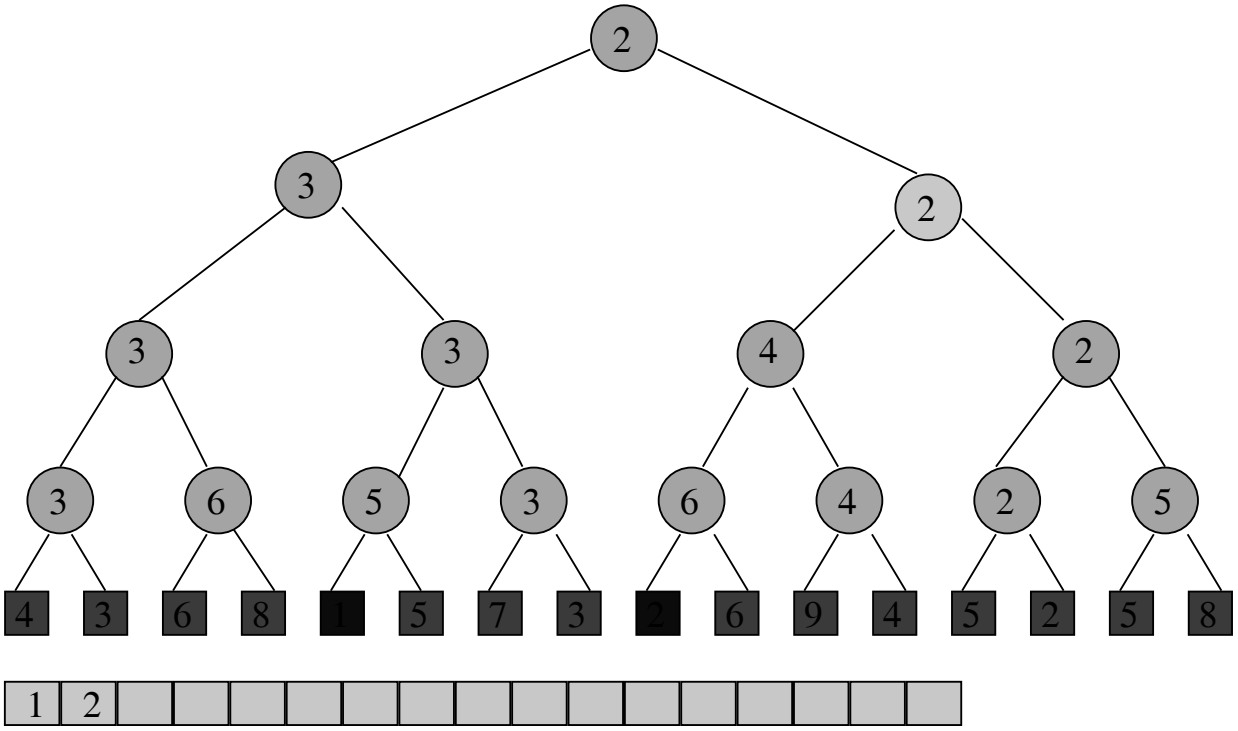
Sorted array.

Sort 16 Numbers



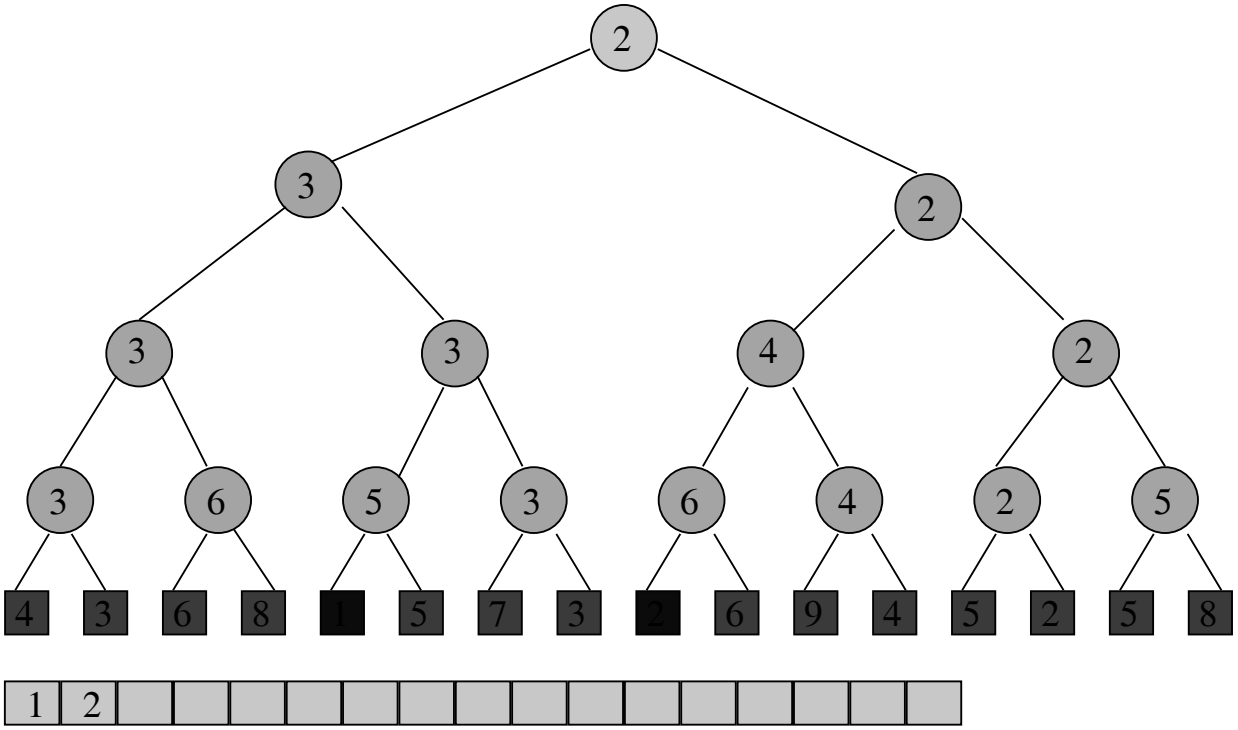
Sorted array.

Sort 16 Numbers



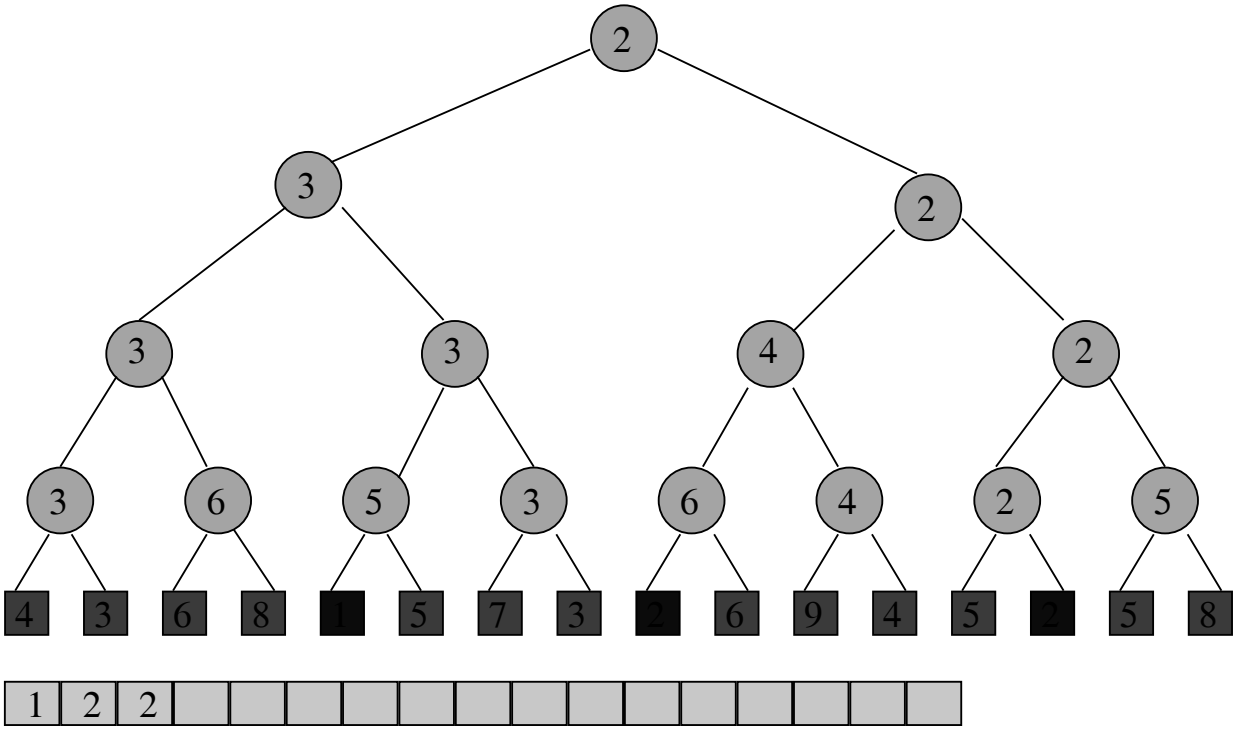
Sorted array.

Sort 16 Numbers



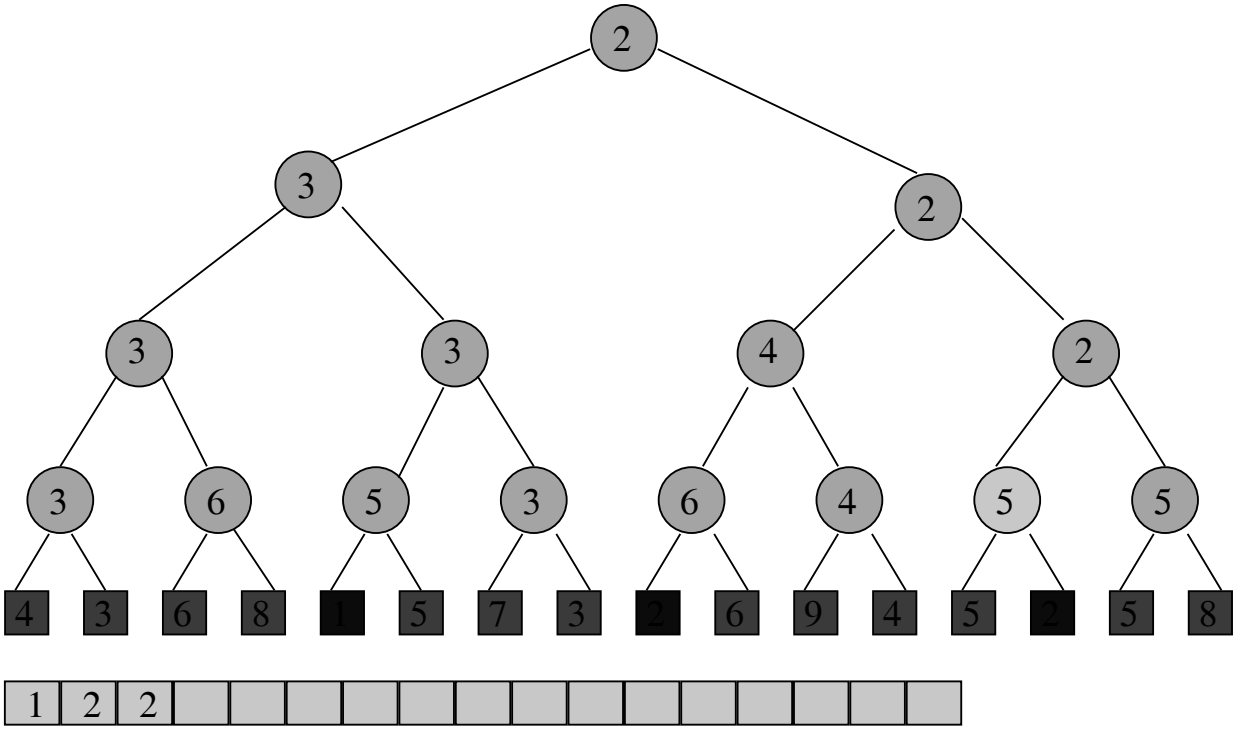
Sorted array.

Sort 16 Numbers



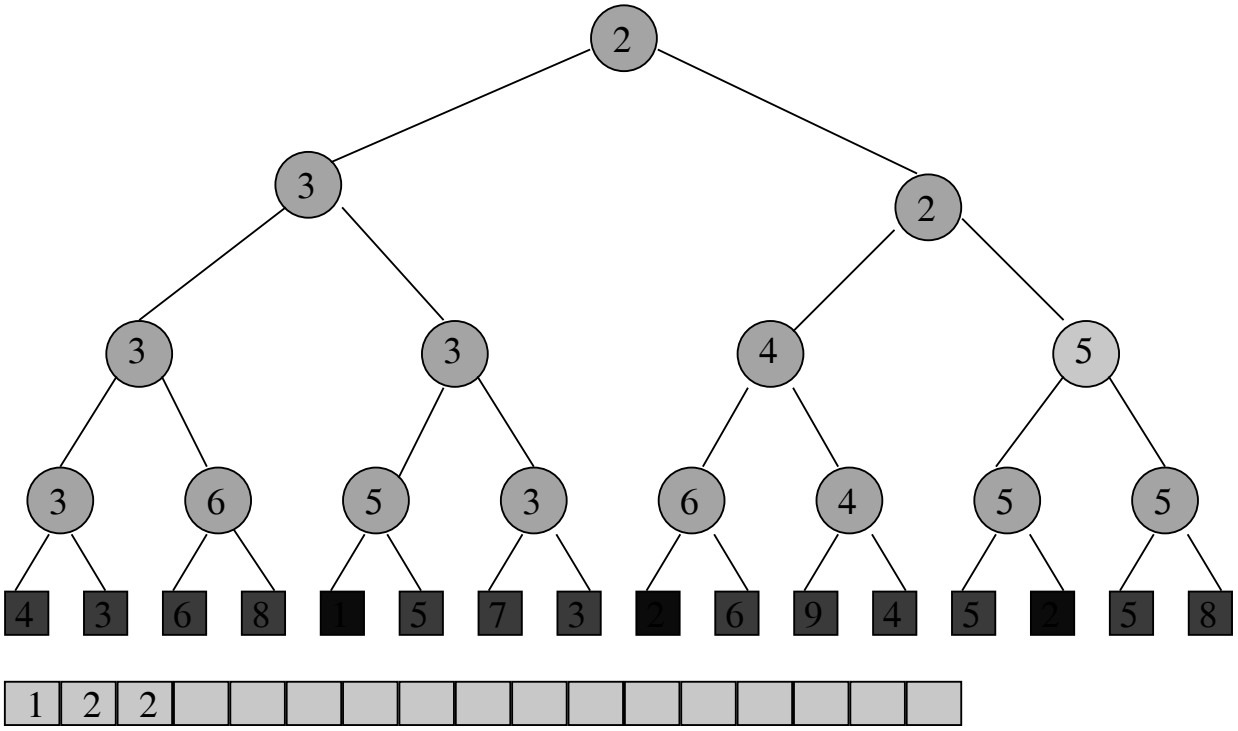
Sorted array.

Sort 16 Numbers



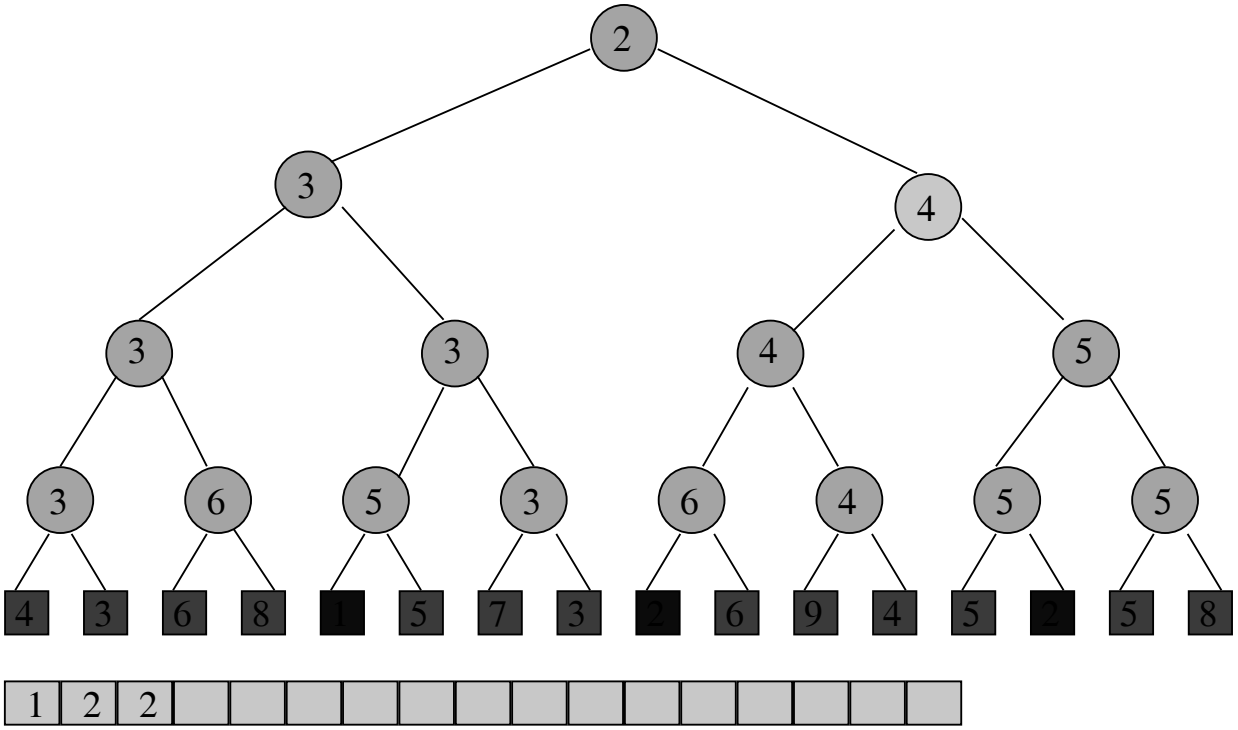
Sorted array.

Sort 16 Numbers



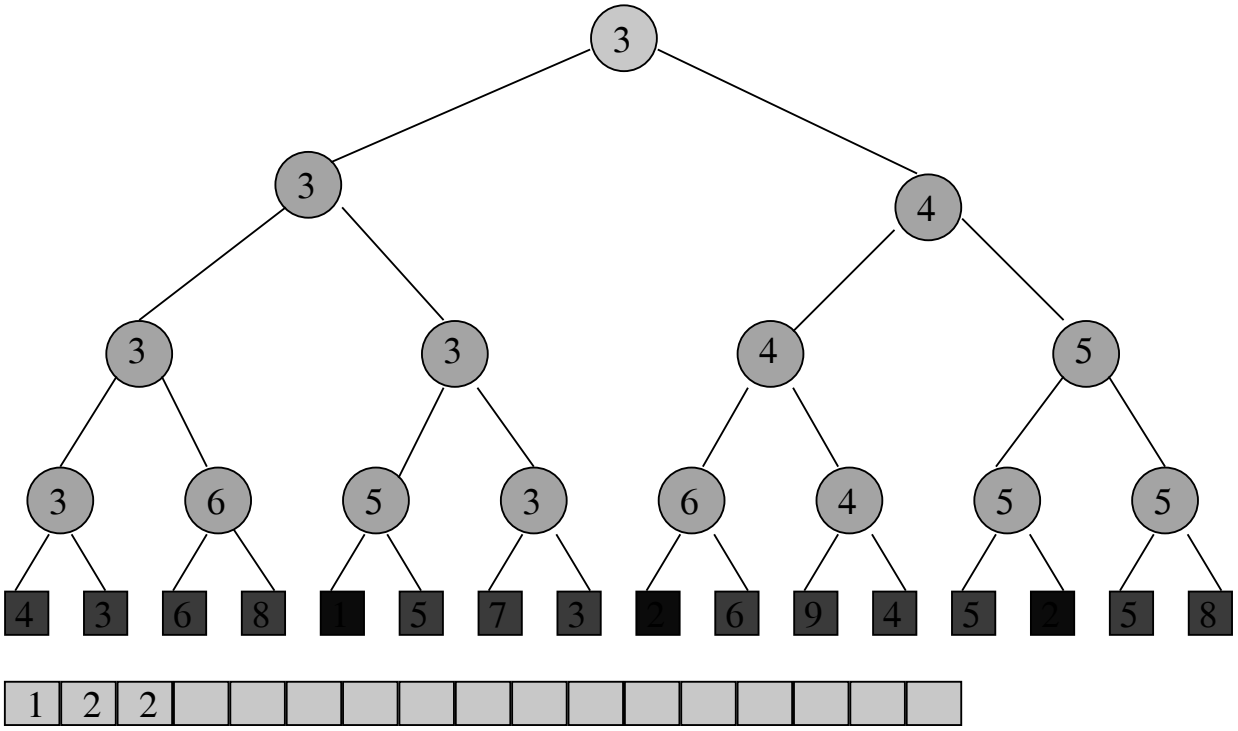
Sorted array.

Sort 16 Numbers



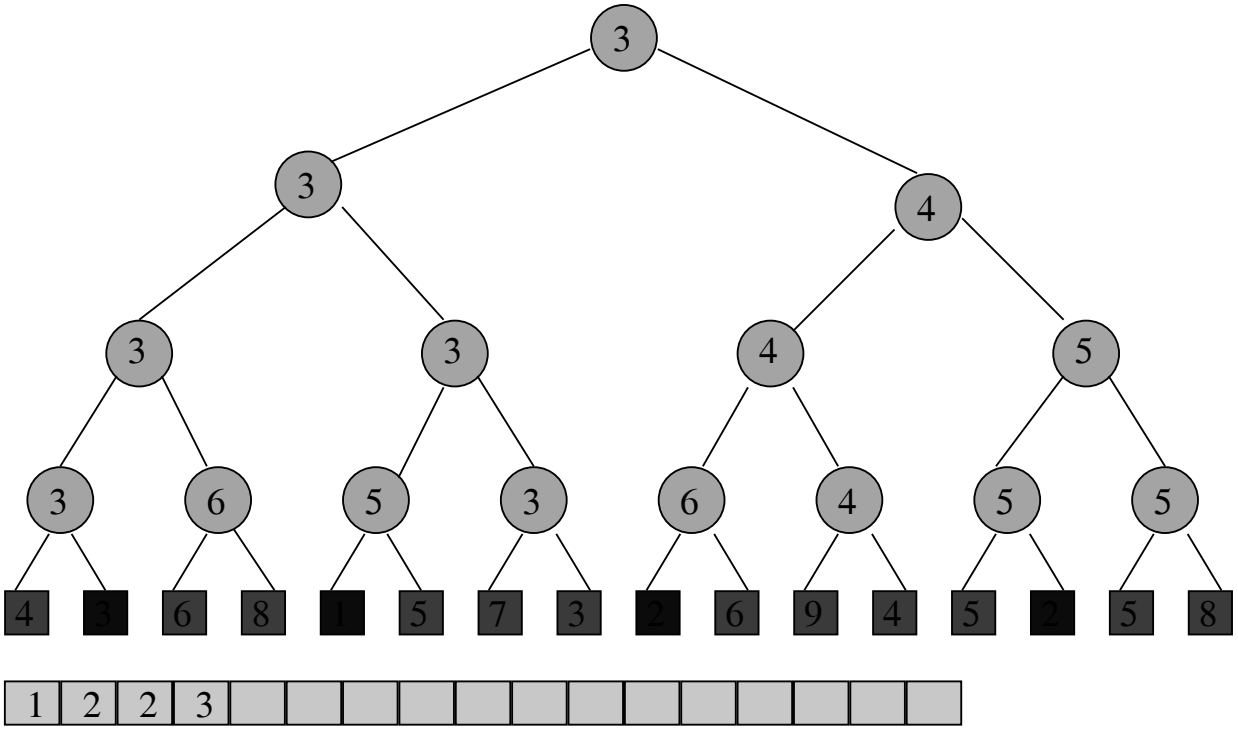
Sorted array.

Sort 16 Numbers



Sorted array.

Sort 16 Numbers



Sorted array.

In Class Exercise

- 請將下一個element 放入 sorted array,並更新winner tree

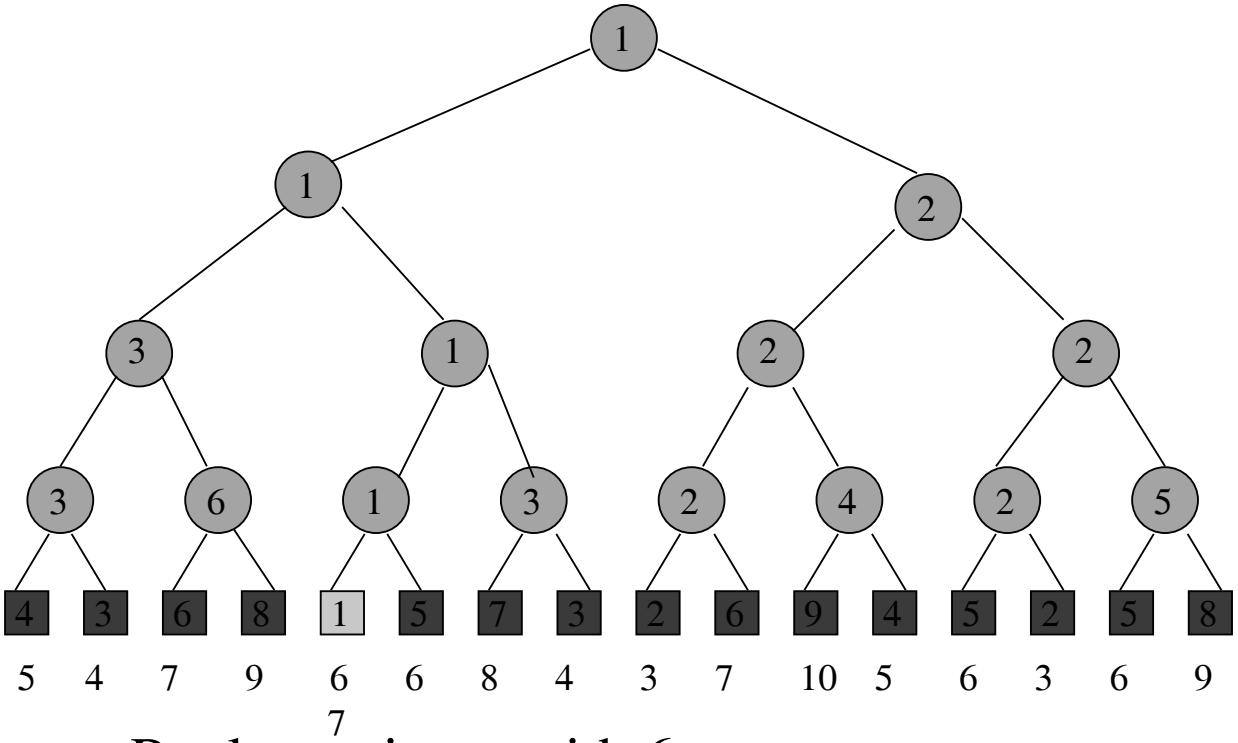
Time To Sort

- Initialize winner tree.
  - O(k) time
- Remove winner and replay.
  - O(log k) time
- Remove winner and replay n times.
  - O(n log k) time
- Total sort time is O(n log k).
- Actually Theta(n log k).

Winner Tree Operations

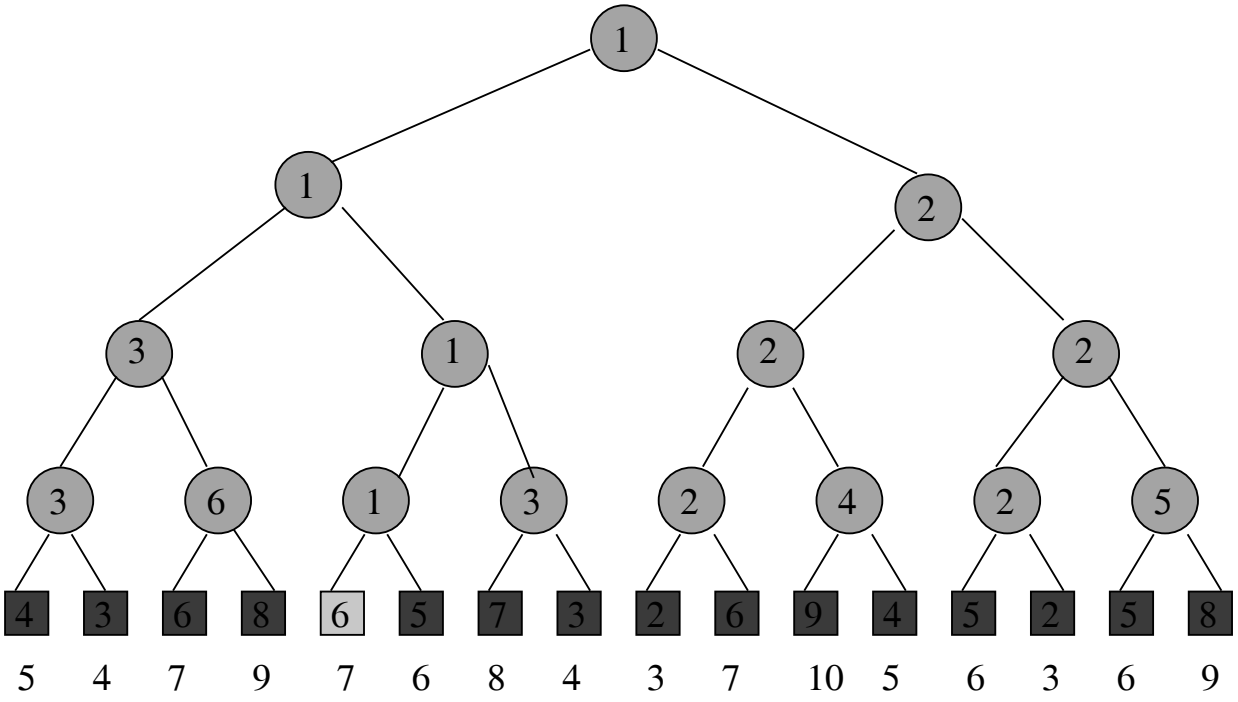
- Initialize
  - O(k) time
- Get winner
  - O(1) time
- Remove/replace winner and replay
  - O(log k) time
  - more precisely Theta(log k)
  - Can be used to merge k-ordered sequences into one.

Replace Winner And Replay



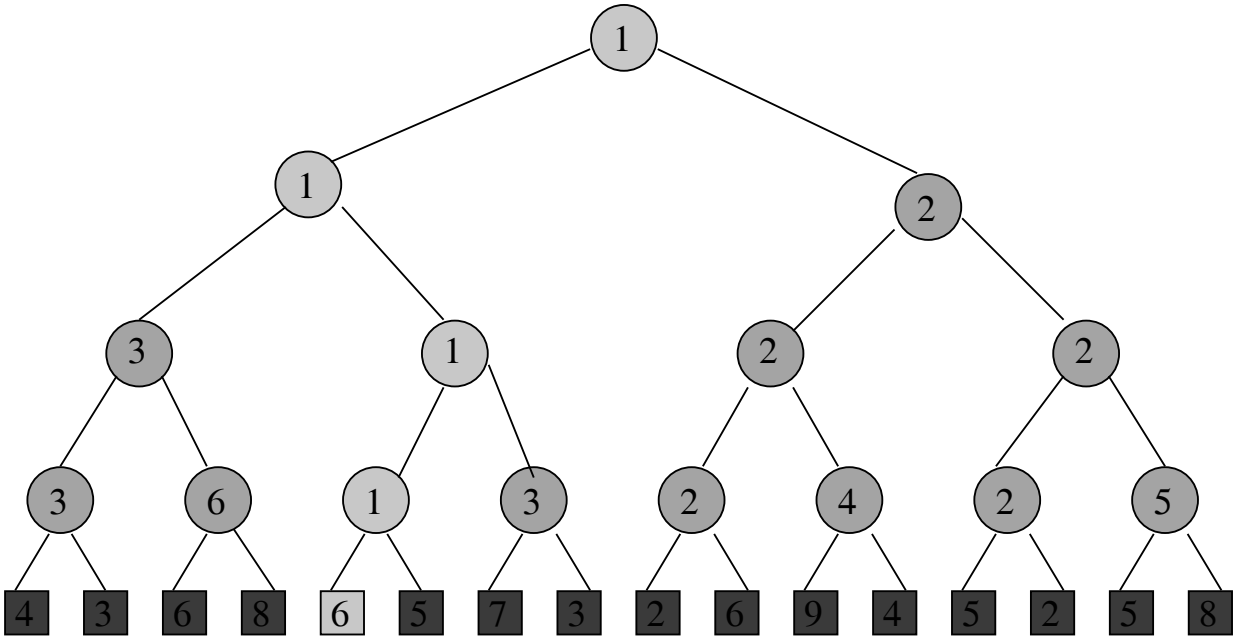
Replace winner with 6.

Replace Winner And Replay



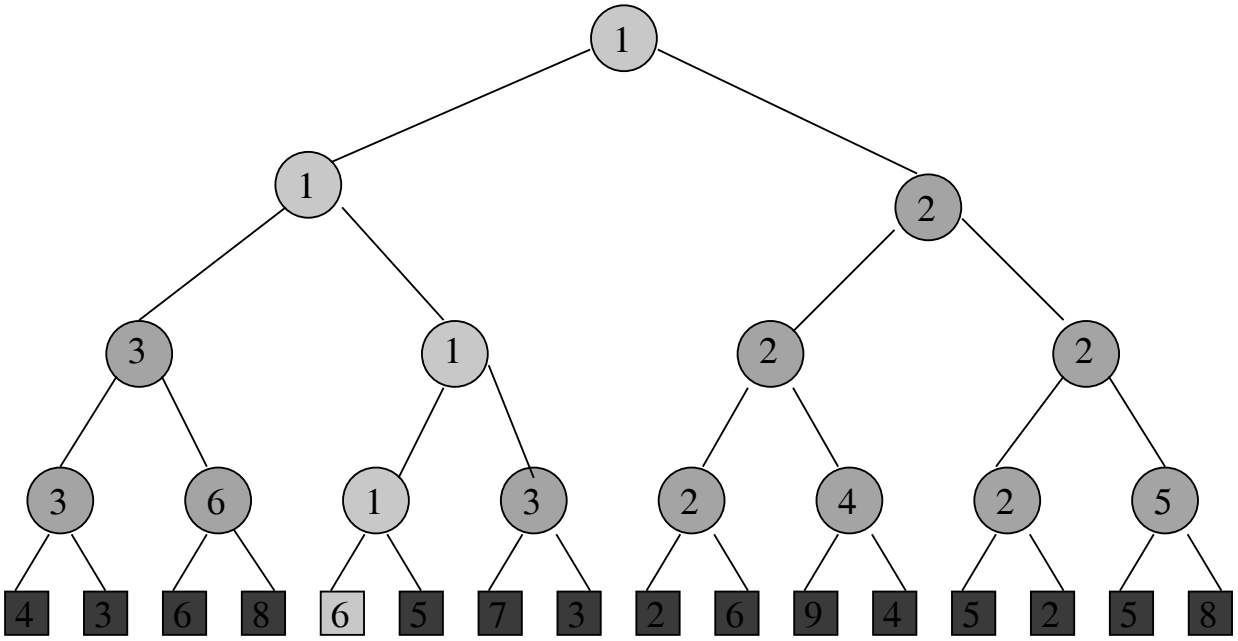
Replay matches on path to root.

Replace Winner And Replay



Replay matches on path to root.

Replace Winner And Replay

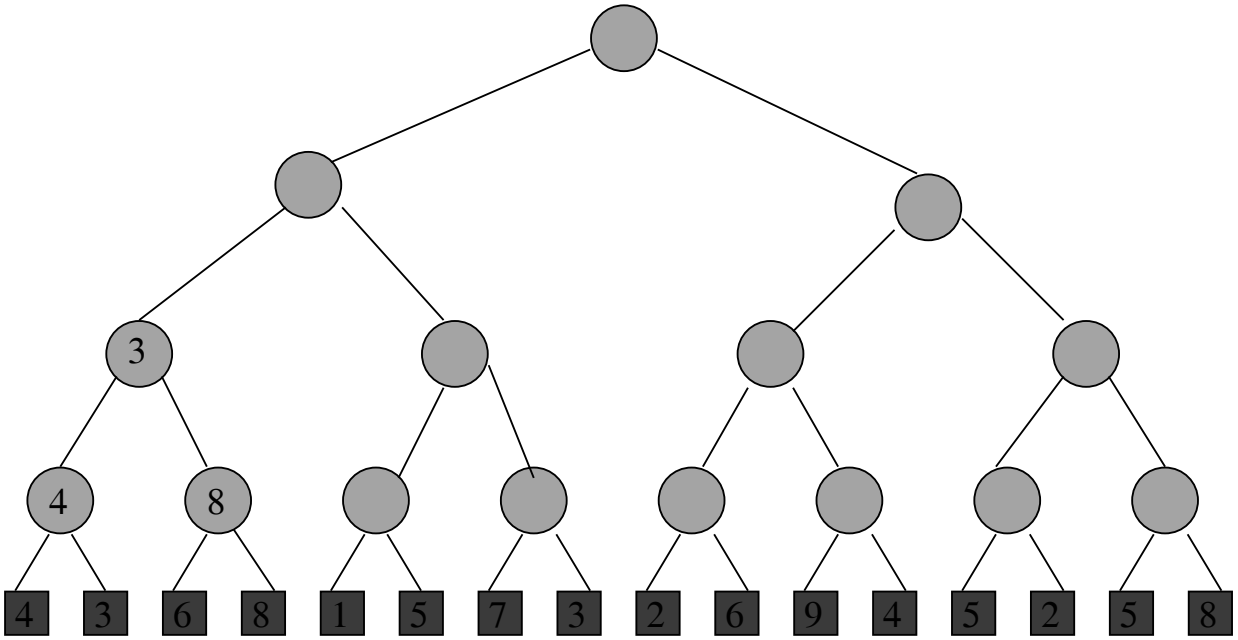


Opponent is player who lost last match played at this node.

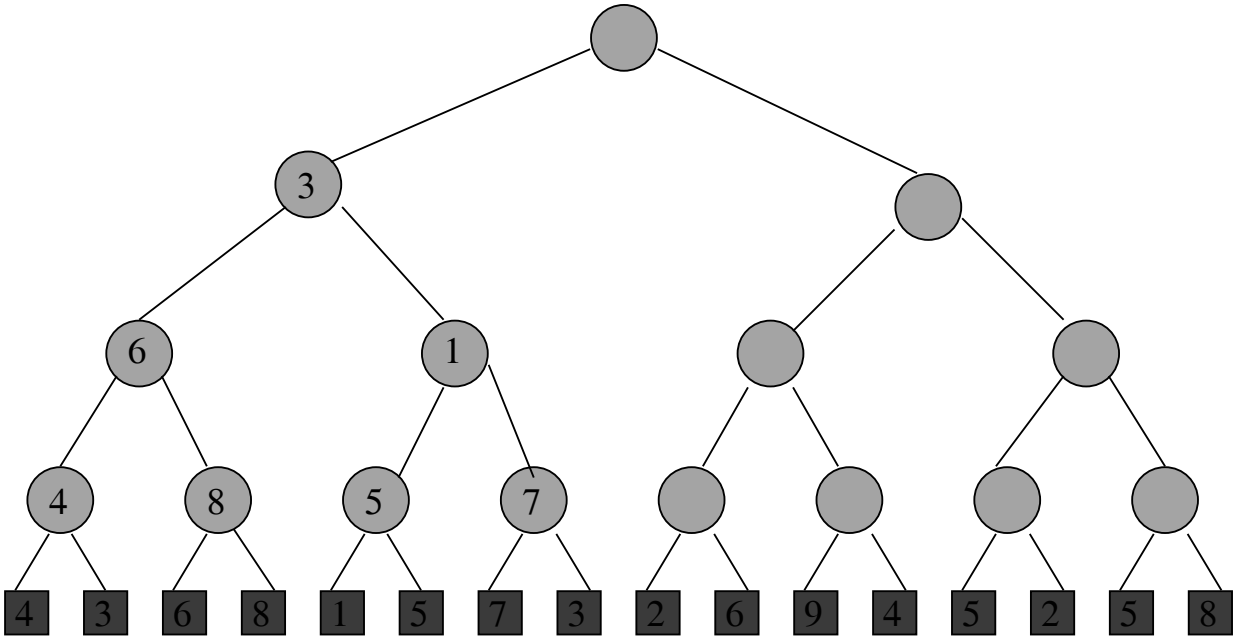
Loser Tree

Each match node stores the match loser rather than the match winner.

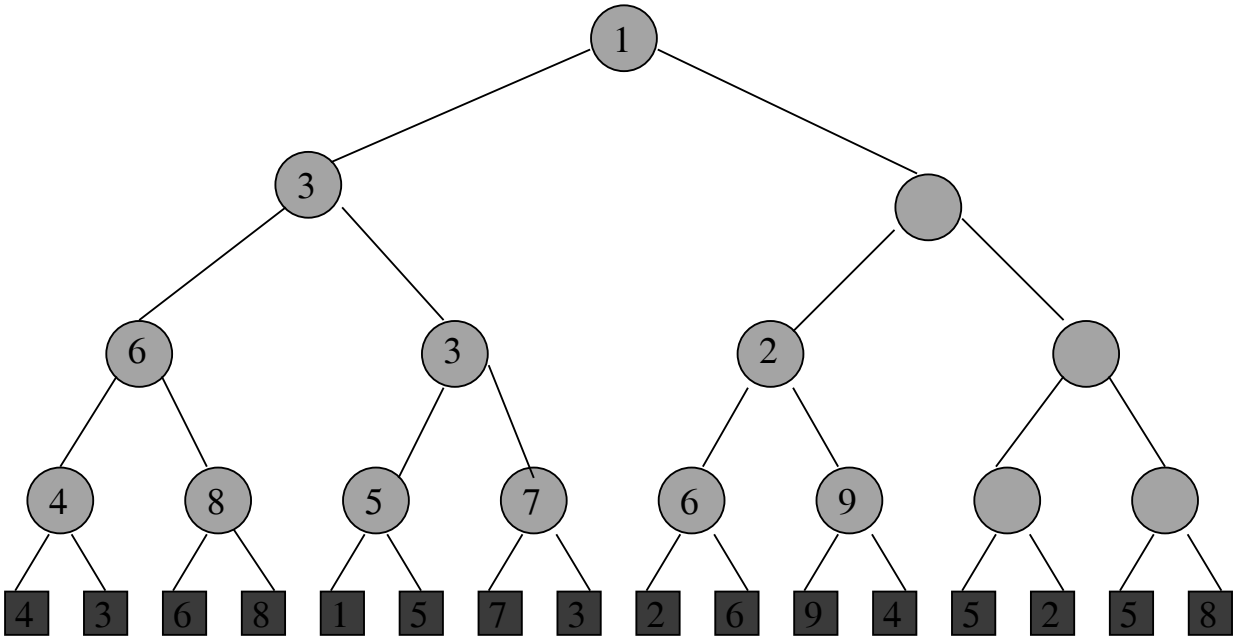
Min Loser Tree For 16 Players



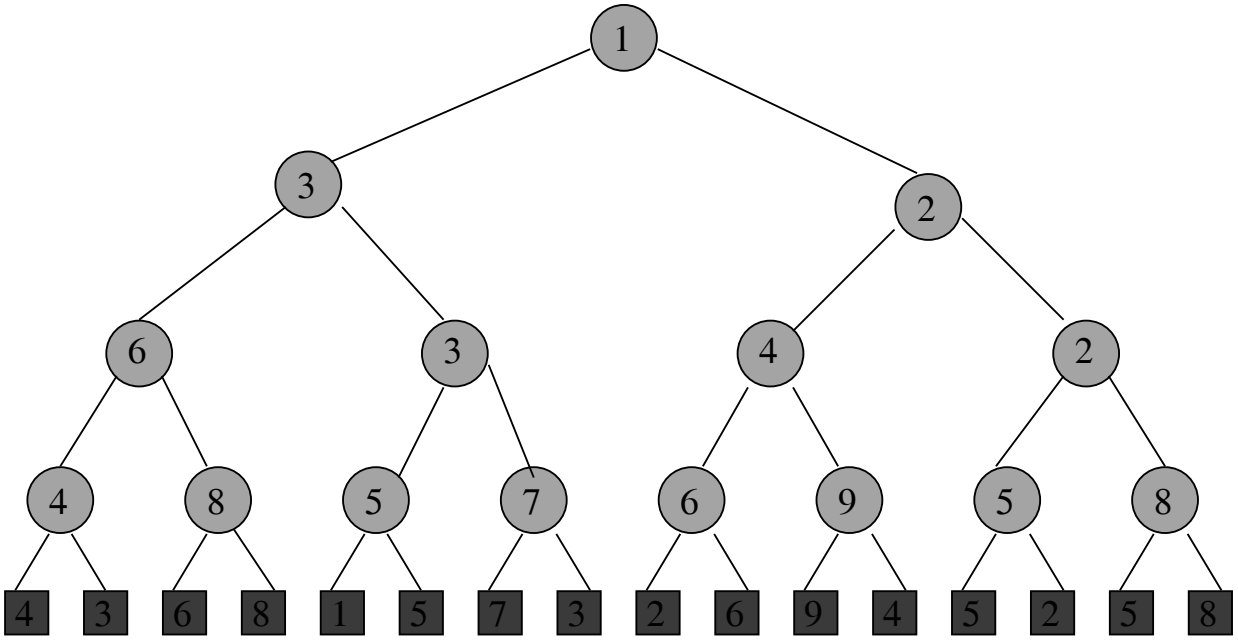
Min Loser Tree For 16 Players



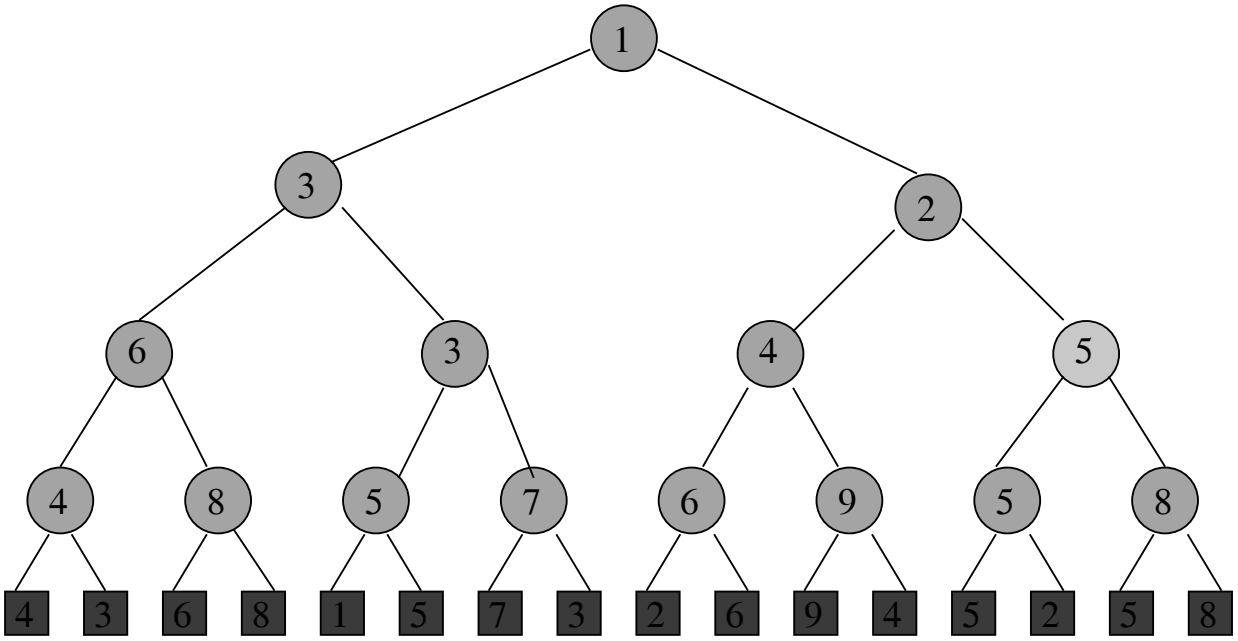
Min Loser Tree For 16 Players



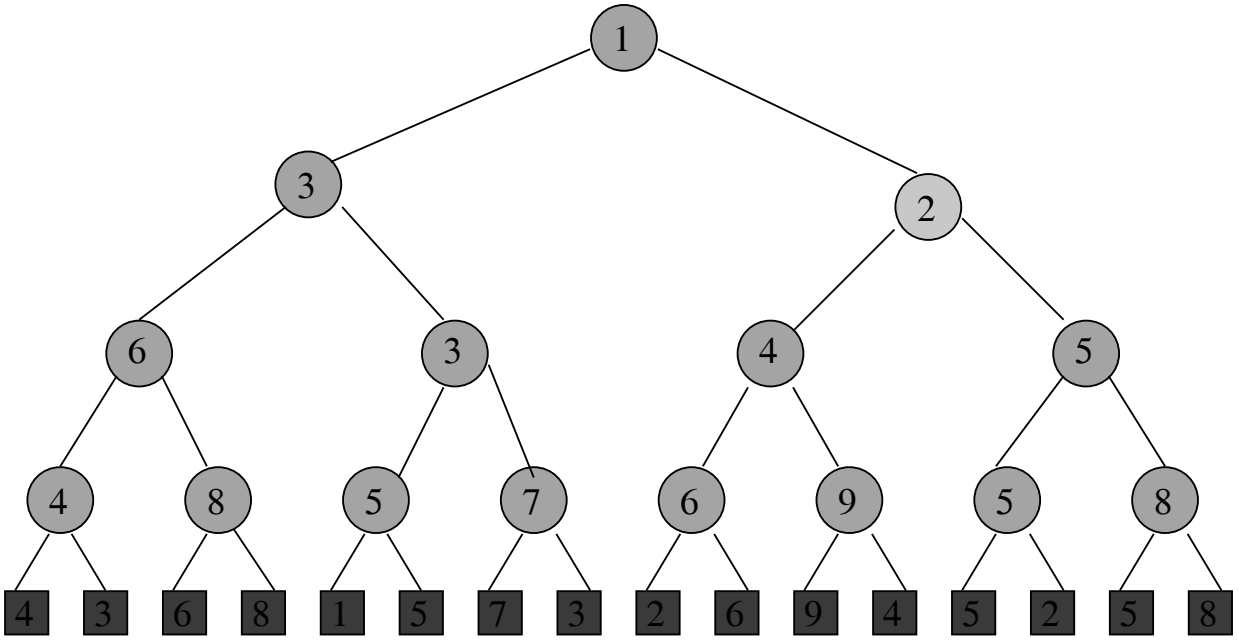
Min Loser Tree For 16 Players



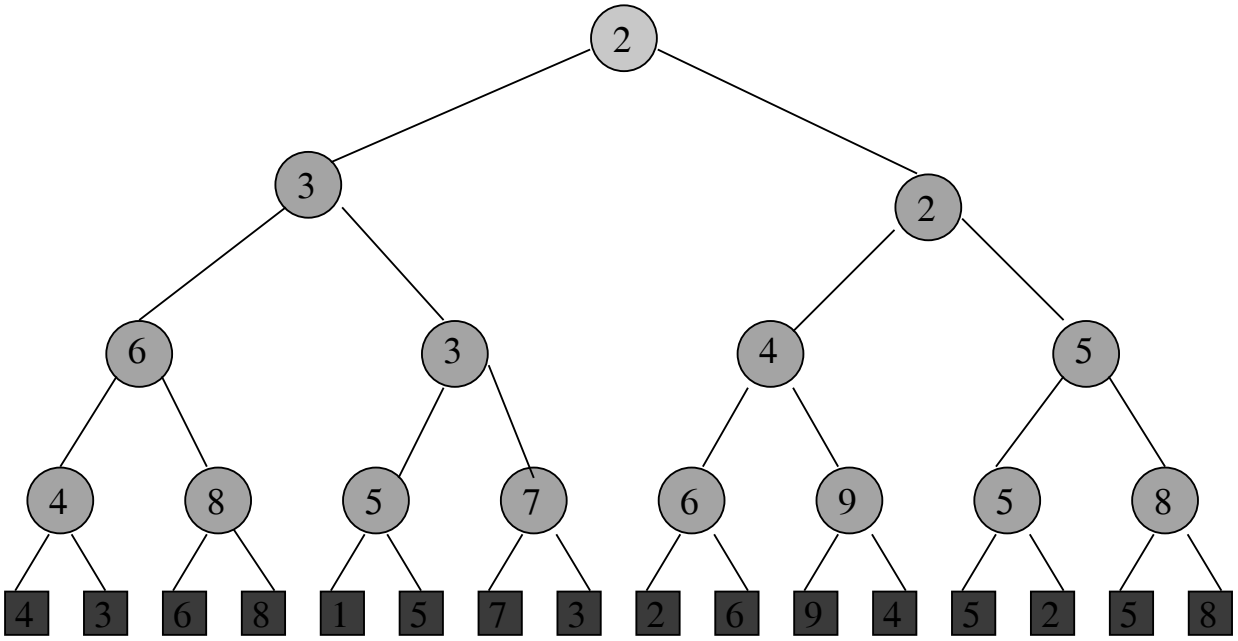
Min Loser Tree For 16 Players

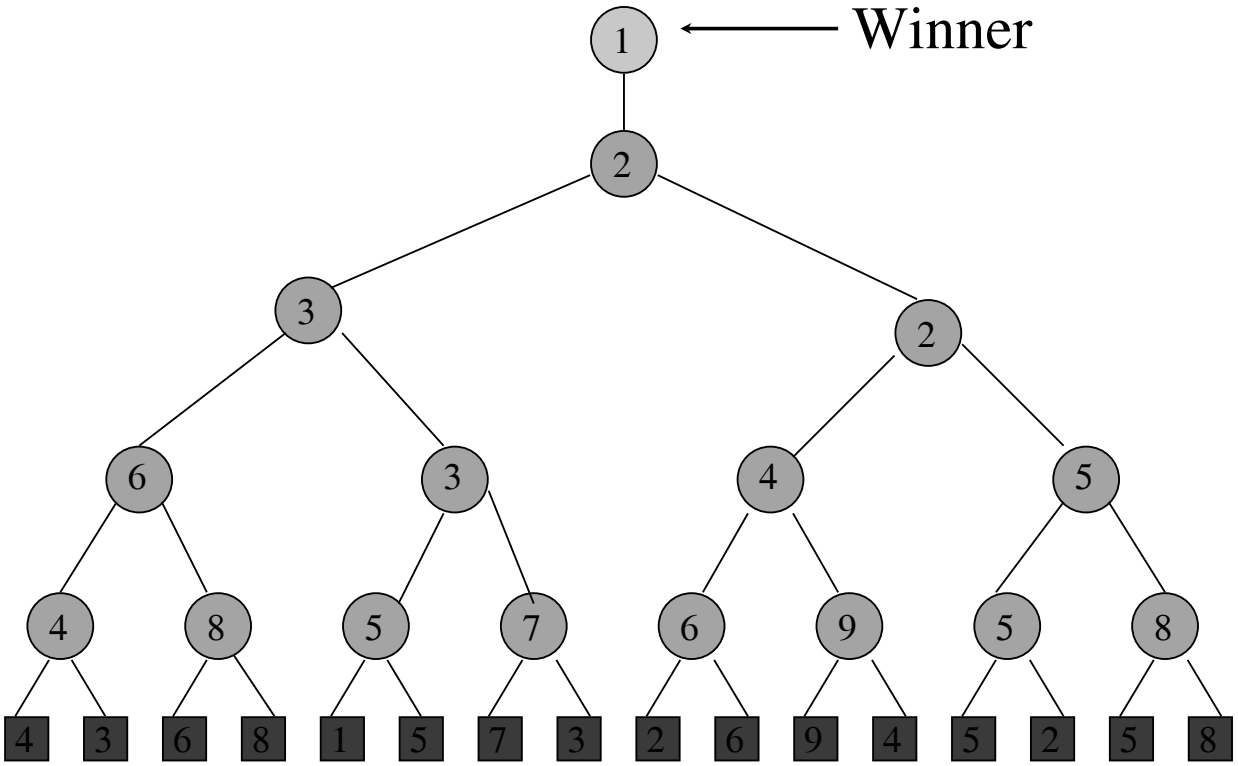


Min Loser Tree For 16 Players



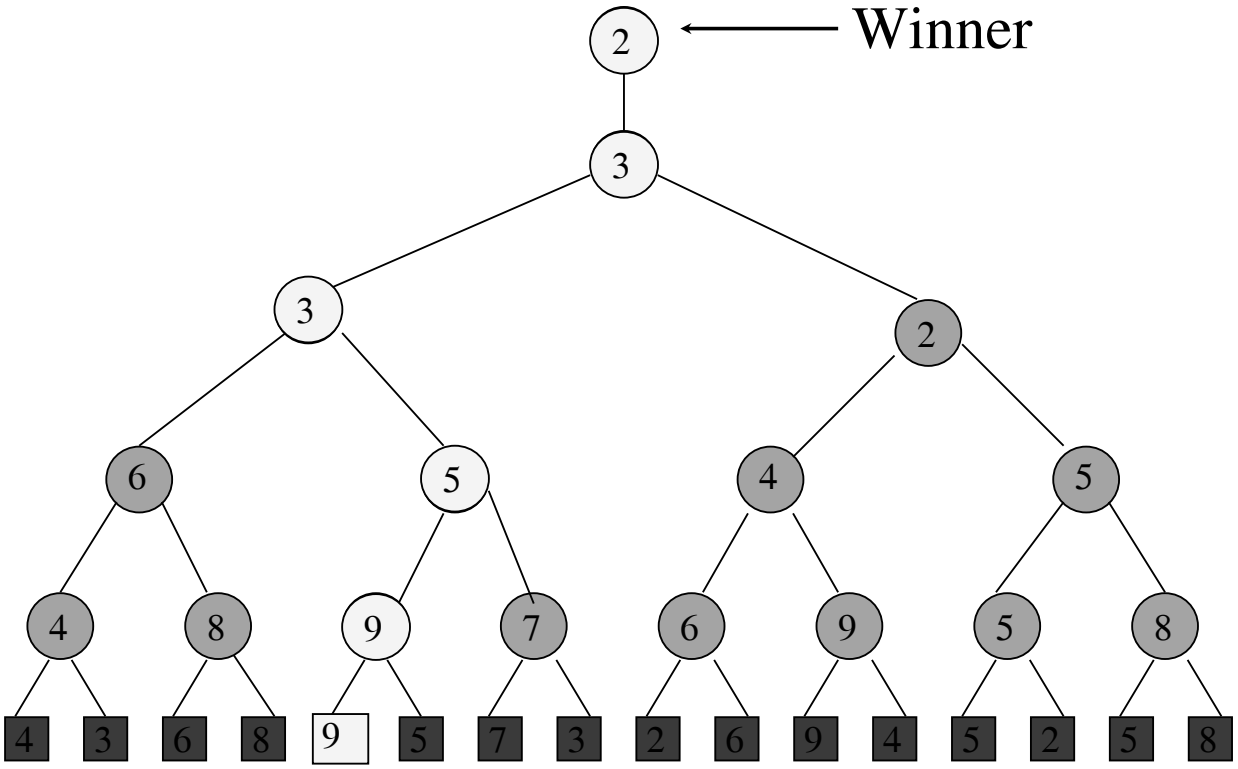
Min Loser Tree For 16 Players





Complexity Of Loser Tree Initialize

- One match at each match node.
- One store of a left child winner.
- Total time is O(k).
- More precisely Theta(k).



Replace winner with 9 and replay matches.

Complexity Of Replay

- One match at each level that has a match node.
- O(log k)
- More precisely Theta(log k).

Homework

- Sec. 5.8 Exercise 2@P301