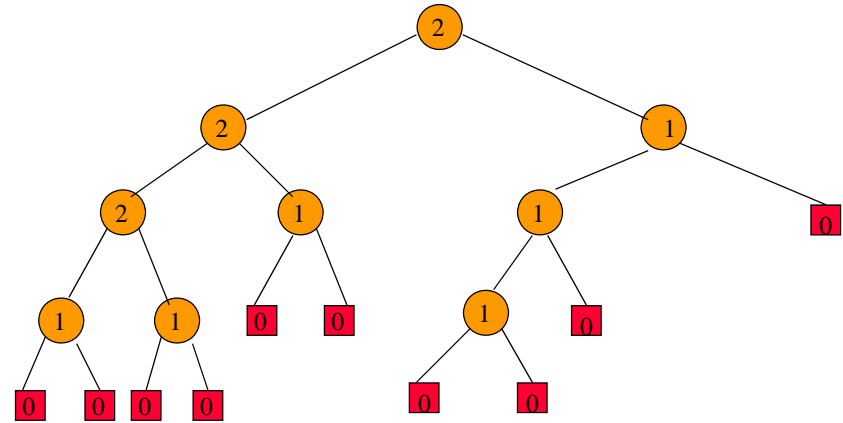


Height Biased Leftist Trees

A binary tree is a (height biased) leftist tree
 iff for every internal node x ,
 $s(\text{leftChild}(x)) \geq s(\text{rightChild}(x))$

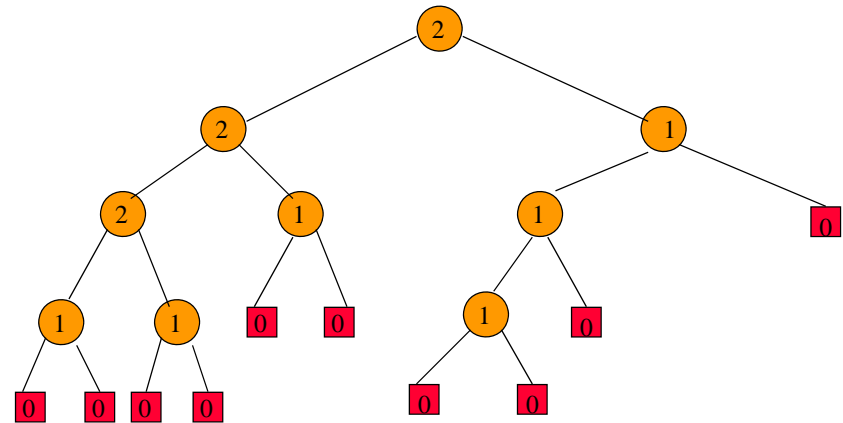
A Leftist Tree



Leftist Trees--Property 1

In a leftist tree, the rightmost path is a
 shortest root to external node path and
 the length of this path is $s(\text{root})$.

A Leftist Tree



Length of rightmost path is 2.

Leftist Trees—Property 2

The number of internal nodes is at least

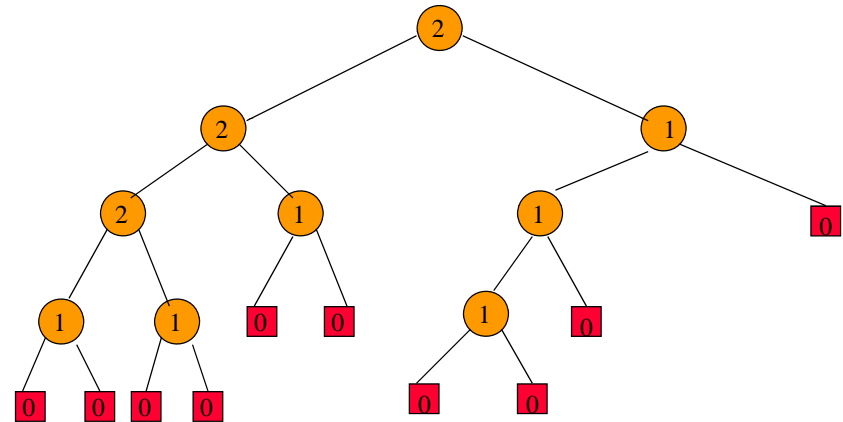
$$2^{s(\text{root})} - 1$$

Because levels 1 through $s(\text{root})$ have no external nodes.

So, $s(\text{root}) \leq \log(n+1)$

13

A Leftist Tree



Levels 1 and 2 have no external nodes.

14

Leftist Trees—Property 3

Length of rightmost path is $O(\log n)$, where

n is the number of nodes in a leftist tree.

Follows from Properties 1 and 2.

15

Leftist Trees As Priority Queues

Min leftist tree ... leftist tree that is a min tree.

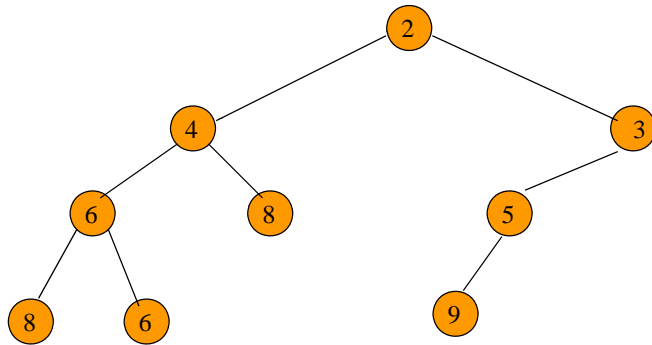
Used as a min priority queue.

Max leftist tree ... leftist tree that is a max tree.

Used as a max priority queue.

16

A Min Leftist Tree



17

Some Min Leftist Tree Operations

empty()

size()

top()

push()

pop()

meld()

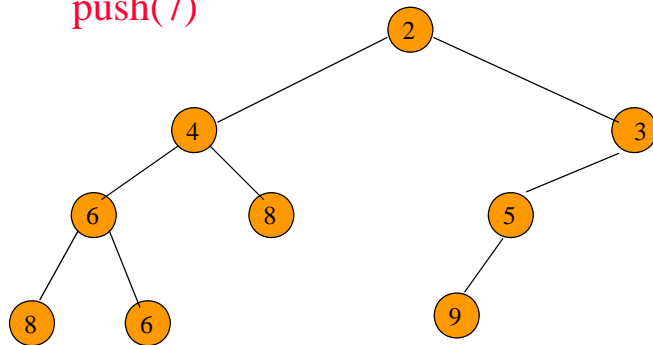
initialize()

push() and pop() use meld().

18

Push Operation

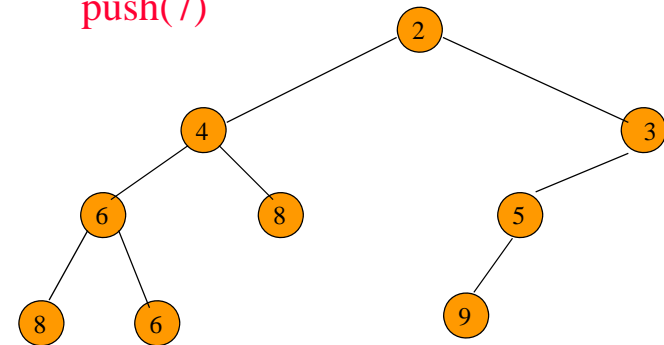
push(7)



19

Push Operation

push(7)



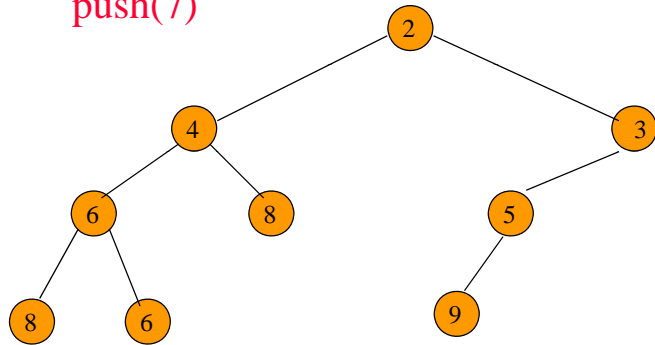
Create a single node min leftist tree.



20

Push Operation

push(7)



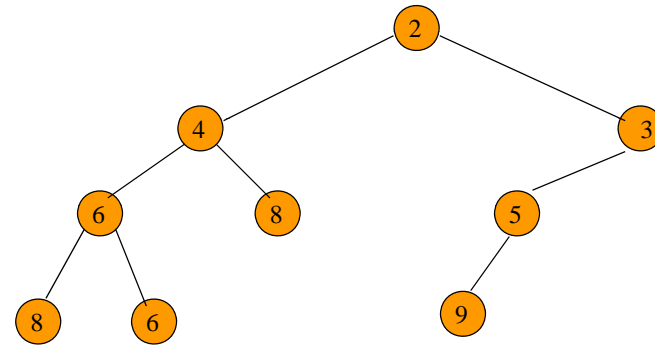
Create a single node min leftist tree.

7

Meld the two min leftist trees. (discussed later)

21

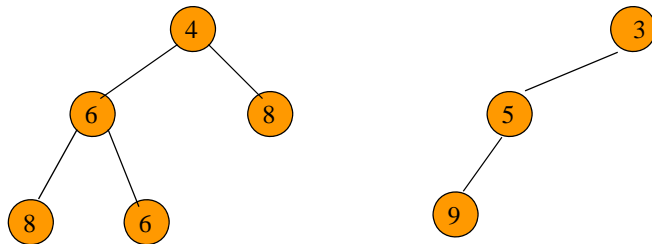
Remove Min (pop)



22

Remove Min (pop)

2

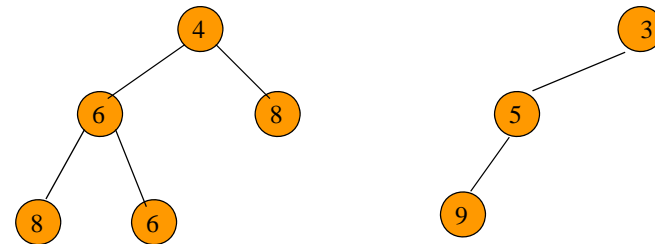


Remove the root.

23

Remove Min (pop)

2

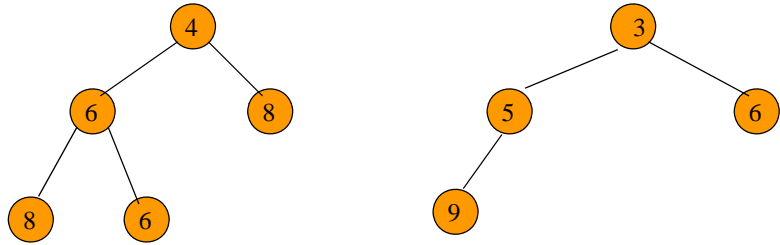


Remove the root.

Meld the two subtrees. (discussed later)

24

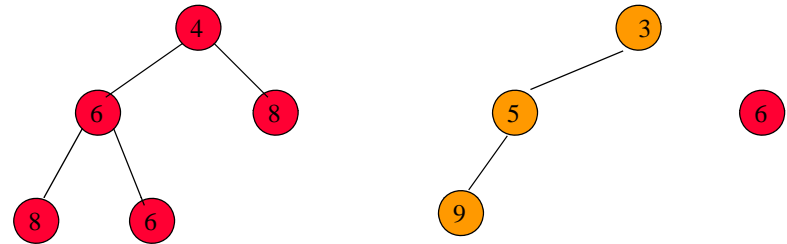
Meld Two Min Leftist Trees



Traverse only the rightmost paths so as to get logarithmic performance.

25

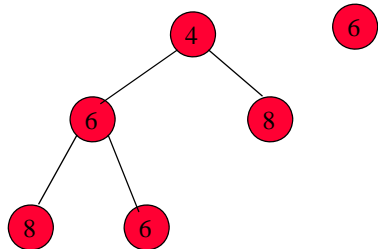
Meld Two Min Leftist Trees



Meld right subtree of tree with smaller root and all of other tree.

26

Meld Two Min Leftist Trees



Meld right subtree of tree with smaller root and all of other tree.

27

Meld Two Min Leftist Trees



Meld right subtree of tree with smaller root and all of other tree.

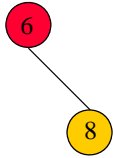
Right subtree of 6 is empty. So, result of melding right subtree of tree with smaller root and other tree is the other tree.

28

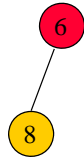
Meld Two Min Leftist Trees



Make melded subtree right subtree of smaller root.

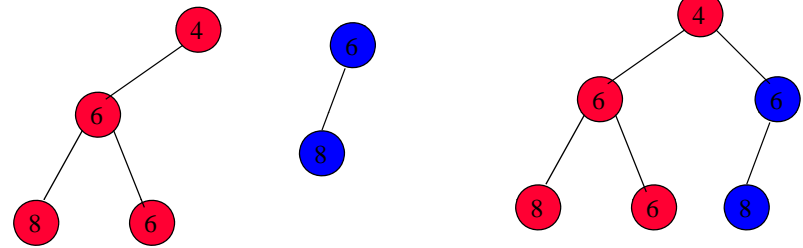


Swap left and right subtree if $s(\text{left}) < s(\text{right})$.



29

Meld Two Min Leftist Trees

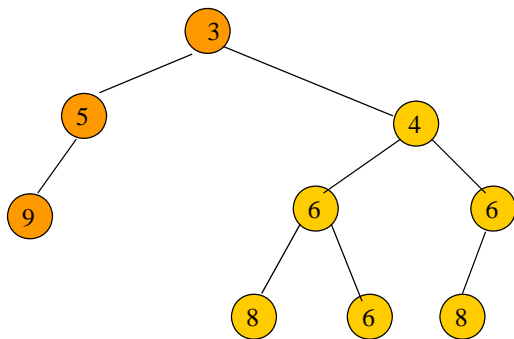


Make melded subtree right subtree of smaller root.

Swap left and right subtree if $s(\text{left}) < s(\text{right})$.

30

Meld Two Min Leftist Trees

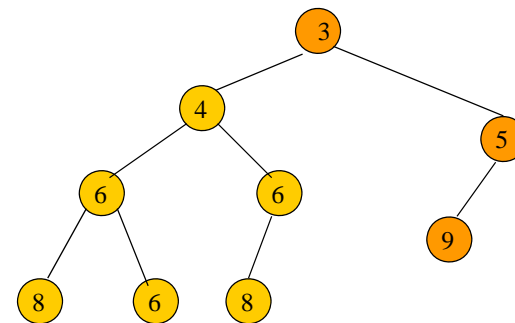


Make melded subtree right subtree of smaller root.

Swap left and right subtree if $s(\text{left}) < s(\text{right})$.

31

Meld Two Min Leftist Trees



32

In Class Exercise

- Remove Min (pop) 3 and show the resulting Leftist tree.

33

Homework

- Sec. 9.2 Exercise 3 @P 500

34