Binary Tree Traversal Methods

- Many binary tree operations are done by performing a traversal of the binary tree.
- Possible Binary Tree Operations:
  - Determine the height.
  - Determine the number of nodes.
  - Make a clone.
  - Evaluate the arithmetic expression represented by a binary tree.
  - …

Binary Tree Traversal Methods

- In a traversal of a binary tree, each element of the binary tree is visited exactly once.
- During the visit of an element, all action (make a clone, display, evaluate the operator, etc.) with respect to this element is taken.

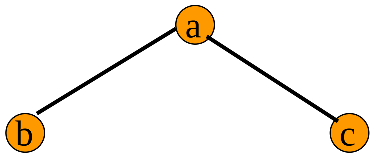
Binary Tree Traversal Methods

- Preorder
- Inorder
- Postorder
- Level order

Preorder Traversal

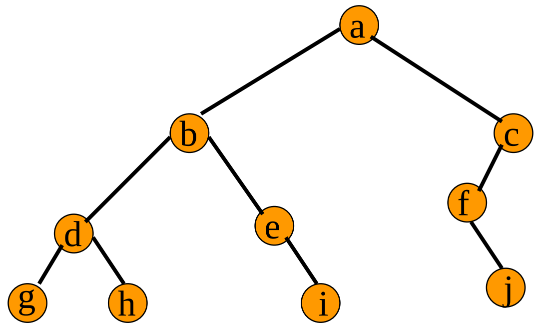
```
template <class T>
void PreOrder(TreeNode<T> *t)
{
   if (t != NULL)
   {
      Visit(t);
      PreOrder(t->leftChild);
      PreOrder(t->rightChild);
   }
}
```

Preorder Example (Visit = print)



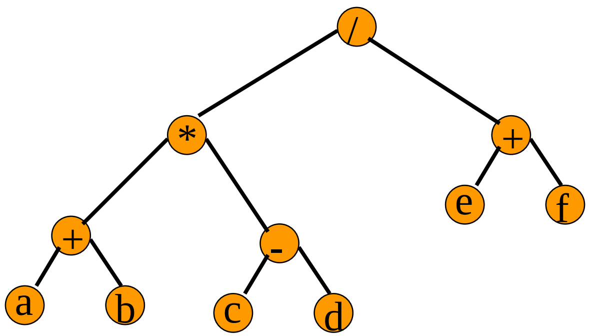
a b c

Preorder Example (Visit = print)



a b d g h e i c f j

Preorder Of Expression Tree



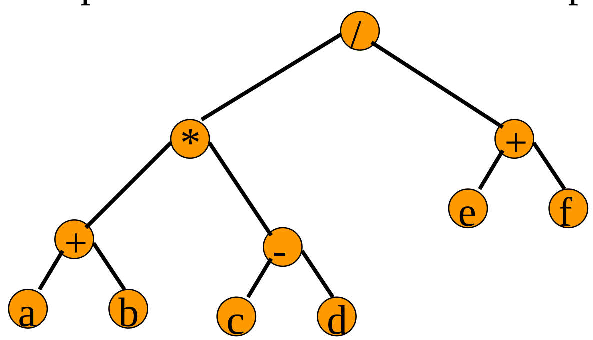
- (a + b) * (c – d) / (e + f)

/ * + a b - c d + e f

Gives prefix form of expression!

Merits Of Binary Tree Form

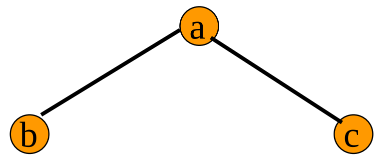
- Left and right operands are easy to visualize.
- Code optimization algorithms work with the binary tree form of an expression.
- Simple recursive evaluation of expression.



Inorder Traversal

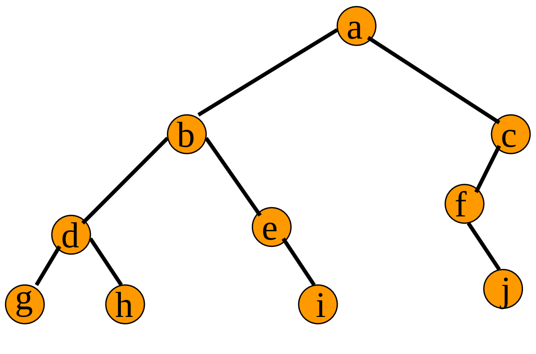
```
template <class T>
void InOrder(TreeNode<T> *t)
{
   if (t != NULL)
   {
      InOrder(t->leftChild);
      Visit(t);
      InOrder(t->rightChild);
   }
}
```

Inorder Example (Visit = print)



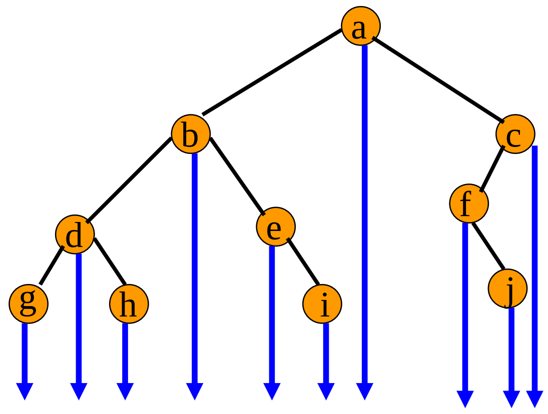
b a c

Inorder Example (Visit = print)



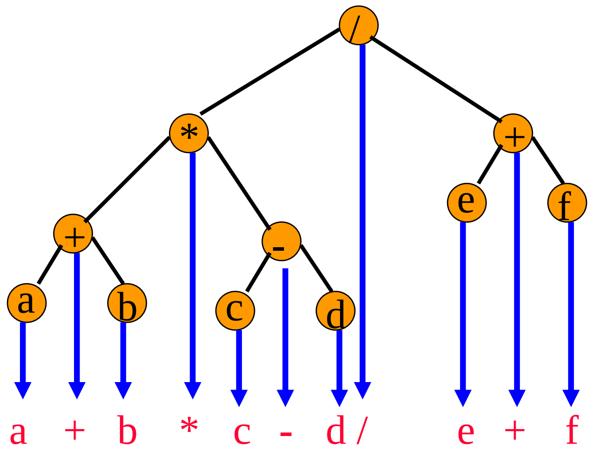
g d h b e i a f j c

Inorder By Projection (Squishing)



g d h b e i a f j c

Inorder Of Expression Tree

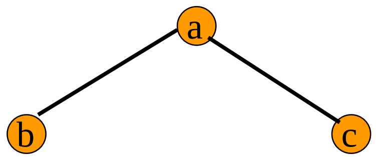


Gives infix form of expression (without parentheses)!

Postorder Traversal

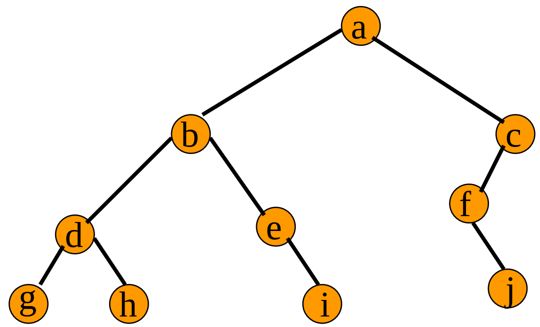
```
template <class T>
void PostOrder(TreeNode<T> *t)
{
   if (t != NULL)
   {
      PostOrder(t->leftChild);
      PostOrder(t->rightChild);
      Visit(t);
   }
}
```

Postorder Example (Visit = print)



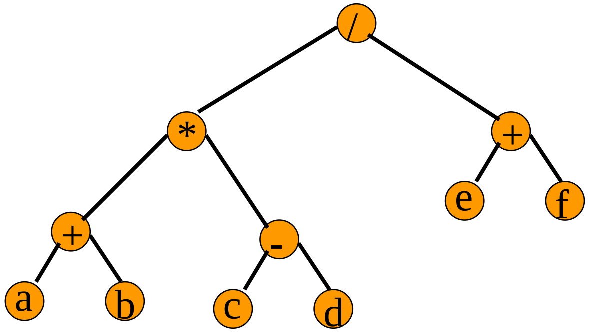
b c a

Postorder Example (Visit = print)



g h d i e b j f c a

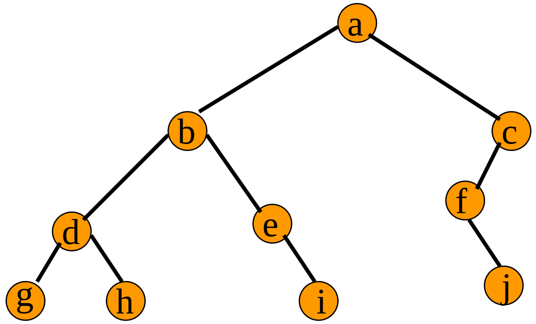
Postorder Of Expression Tree



a b + c d - * e f + /

Gives postfix form of expression!

Traversal Applications



- Make a clone.
- Determine height.
- Determine number of nodes.

Level Order

Let t be the tree root.

while (t != NULL)

{

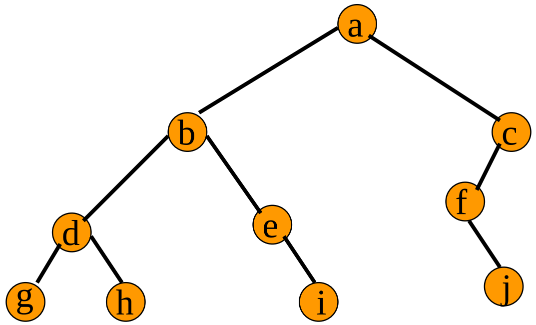
visit t and put its children on a FIFO queue;

if FIFO queue is empty, set t = NULL;

otherwise, pop a node from the FIFO queue and call it t;

}

Level-Order Example (Visit = print)



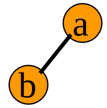
a b c d e f g h i j

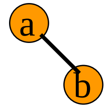
Binary Tree Construction

- Suppose that the elements in a binary tree are distinct.
- Can you construct the binary tree from which a given traversal sequence came?
- When a traversal sequence has more than one element, the binary tree is not uniquely defined.
- Therefore, the tree from which the sequence was obtained cannot be reconstructed uniquely.

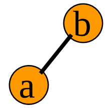
Some Examples

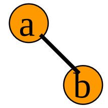
preorder
= ab



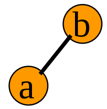


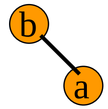
inorder
= ab



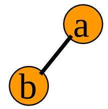


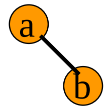
postorder
= ab





level order
= ab



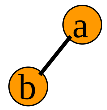


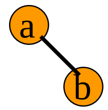
Binary Tree Construction

- Can you construct the binary tree, given two traversal sequences?
- Depends on which two sequences are given.

Preorder And Postorder

preorder = ab
postorder = ba

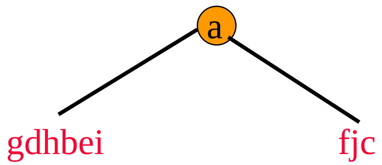




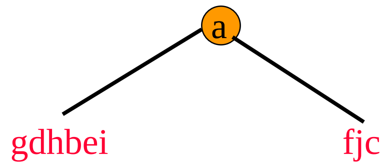
- Preorder and postorder do not uniquely define a binary tree.
- Nor do preorder and level order (same example).
- Nor do postorder and level order (same example).

Inorder And Preorder

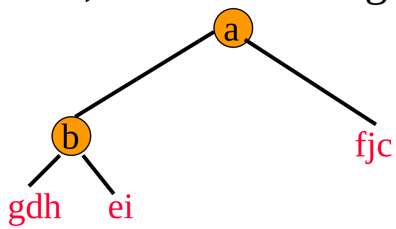
- inorder = g d h b e i a f j c
- preorder = a b d g h e i c f j
- Scan the preorder left to right using the inorder to separate left and right subtrees.
- a is the root of the tree; gdhbei are in the left subtree; fjc are in the right subtree.



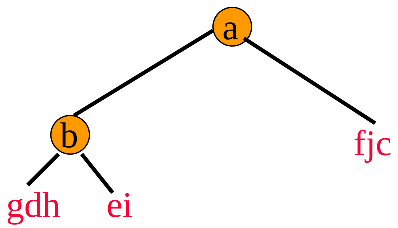
Inorder And Preorder



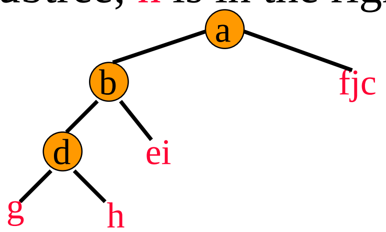
- preorder = a b d g h e i c f j
- b is the next root; gdh are in the left subtree; ei are in the right subtree.



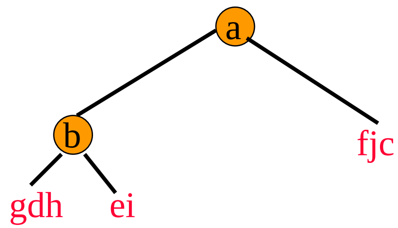
Inorder And Preorder



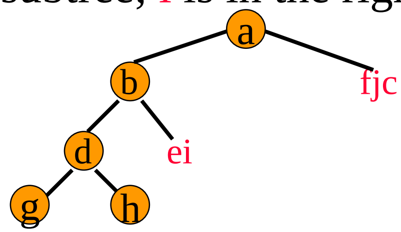
- preorder = a b d g h e i c f j
- d is the next root; g is in the left subtree; h is in the right subtree.



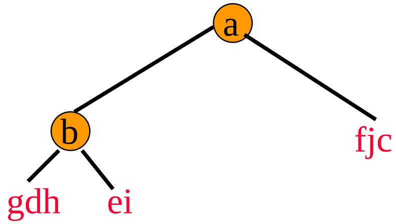
Inorder And Preorder



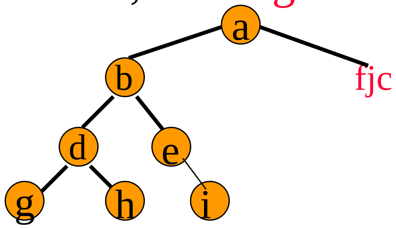
- preorder = a b d g h e i c f j
- e is the next root; nothing is in the left subtree; i is in the right subtree.



Inorder And Preorder



- preorder = a b d g h e i c f j
- c is the next root; fj is in the left subtree; nothing is in the right subtree.



Inorder And Postorder

- Scan postorder from right to left using inorder to separate left and right subtrees.
- inorder = g d h b e i a f j c
- postorder = g h d i e b j f c a
- Tree root is a; gdhbei are in left subtree; fjc are in right subtree.

In Class Exercise

- Determine the tree
  - inorder = g d h b e i a f j c
  - postorder = g h d i e b j f c a

Inorder And Level Order

- Scan level order from left to right using inorder to separate left and right subtrees.
- inorder = g d h b e i a f j c
- level order = a b c d e f g h i j
- Tree root is a; gdhbei are in left subtree; fjc are in right subtree.

Homework

- Sec. 5.3 Exercise 10 @P 267
  - Remark: ADT 5.1 is defined @ P252