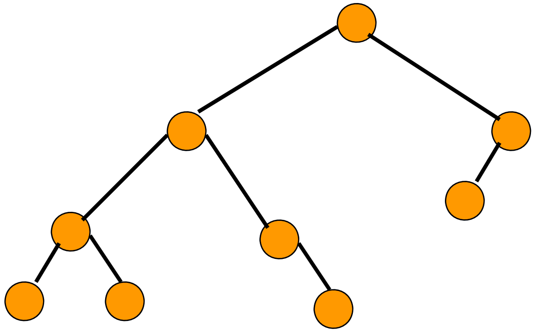
Binary Tree Properties & Representation

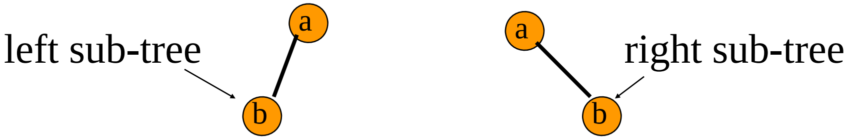


Binary Tree

- Finite (possibly empty) collection of elements.
- A nonempty binary tree has a root element.
- The remaining elements (if any) are partitioned into two binary trees.
- These are called the left and right subtrees of the binary tree.

Differences Between A Tree & A Binary Tree

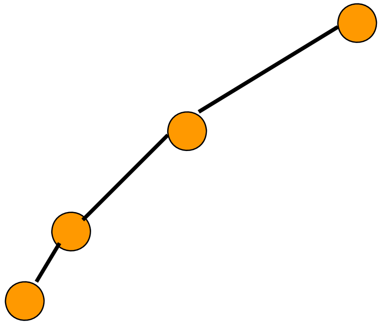
- No node in a binary tree may have a degree more than 2, whereas there is no limit on the degree of a node in a tree.
- The subtrees of a binary tree are ordered; those of a tree are not ordered.



- Are different when viewed as binary trees.
- Are the same when viewed as trees.

Minimum Number Of Nodes

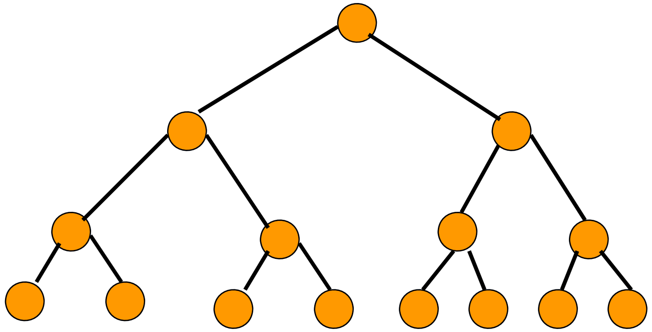
- Minimum number of nodes in a binary tree whose height is h.
- At least one node at each of first h levels.



minimum number of nodes is h

Maximum Number Of Nodes

- All possible nodes at first h levels are present.



Maximum number of nodes

$= 1 + 2 + 4 + 8 + \ldots + 2^{h-1}$

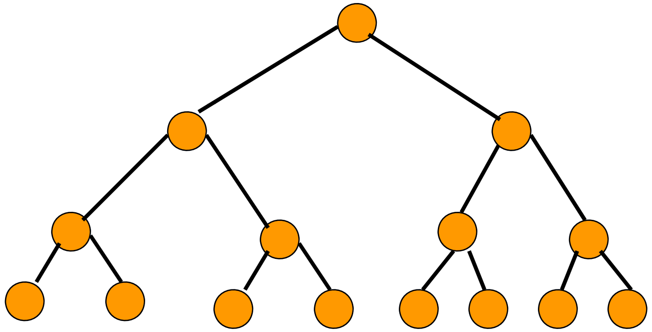
$= 2^h - 1$

Number Of Nodes & Height

- Let n be the number of nodes in a binary tree whose height is h.
- $h <= n <= 2^h – 1$
- $\log_2(n+1) <= h <= n$

Full Binary Tree

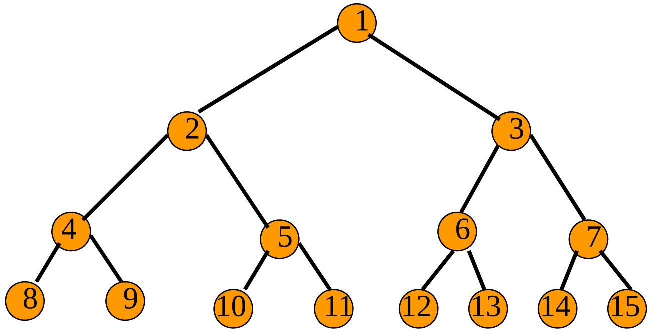
- A full binary tree of a given height h has $2^h – 1$ nodes.



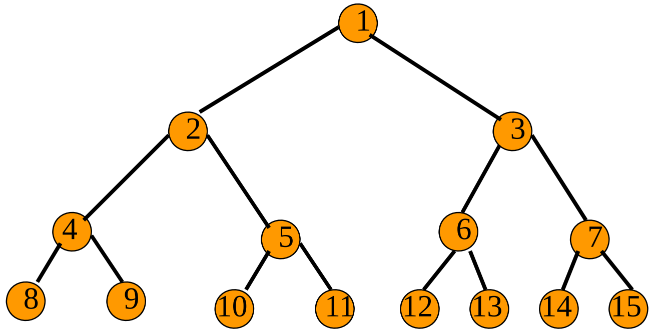
Height 4 full binary tree.

Numbering Nodes In A Full Binary Tree

- Number the nodes 1 through $2^h – 1$.
- Number by levels from top to bottom.
- Within a level number from left to right.

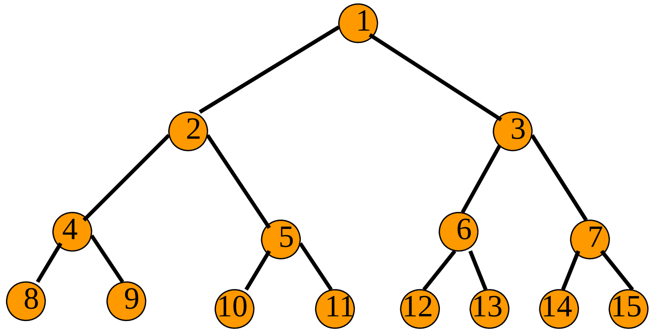


Node Number Properties



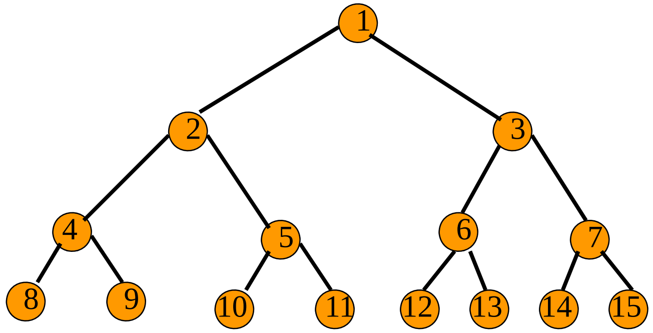
- Parent of node i is node i / 2, unless i = 1.
- Node 1 is the root and has no parent.

Node Number Properties



- Left child of node i is node 2i, unless 2i > n, where n is the number of nodes.
- If 2i > n, node i has no left child.

Node Number Properties

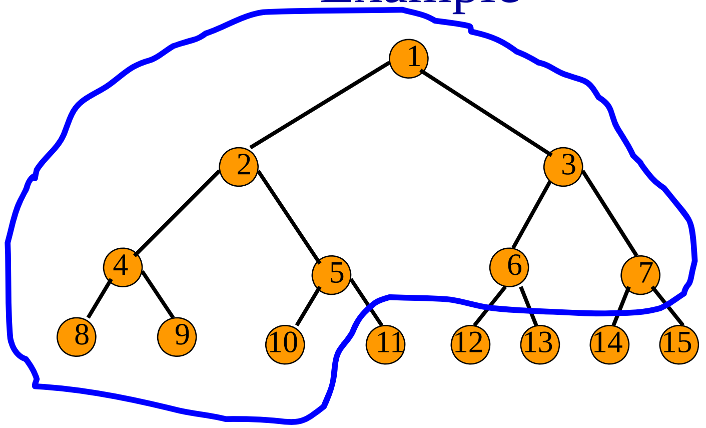


- Right child of node i is node 2i+1, unless 2i+1 > n, where n is the number of nodes.
- If 2i+1 > n, node i has no right child.

Complete Binary Tree With n Nodes

- Start with a full binary tree that has at least n nodes.
- Number the nodes as described earlier.
- The binary tree defined by the nodes numbered 1 through n is the unique n node complete binary tree.

Example



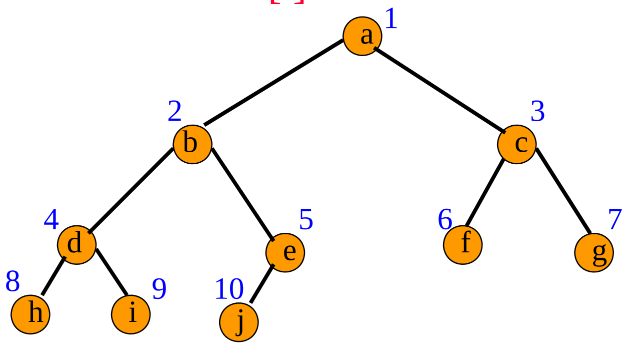
- Complete binary tree with 10 nodes.

Binary Tree Representation

- Array representation.
- Linked representation.

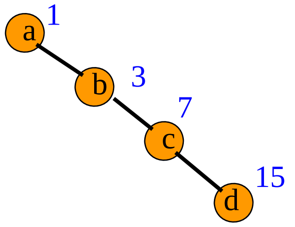
Array Representation

- Number the nodes using the numbering scheme for a full binary tree. The node that is numbered i is stored in tree[i].



tree[]		a	b	c	d	e	f	g	h	i	j
	0					5					10

Right-Skewed Binary Tree

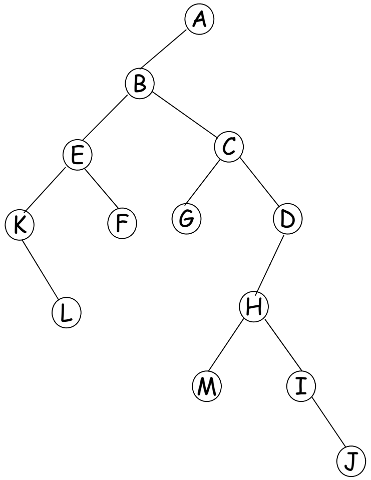


tree[]		a	-	b	-	-	-	c	-	-	-	-	-	-	-	d
	0					5					10					15

Waste memory!

- An n node binary tree needs an array whose length is between n+1 and 2^n.

In Class Exercise
Array representation for the following tree



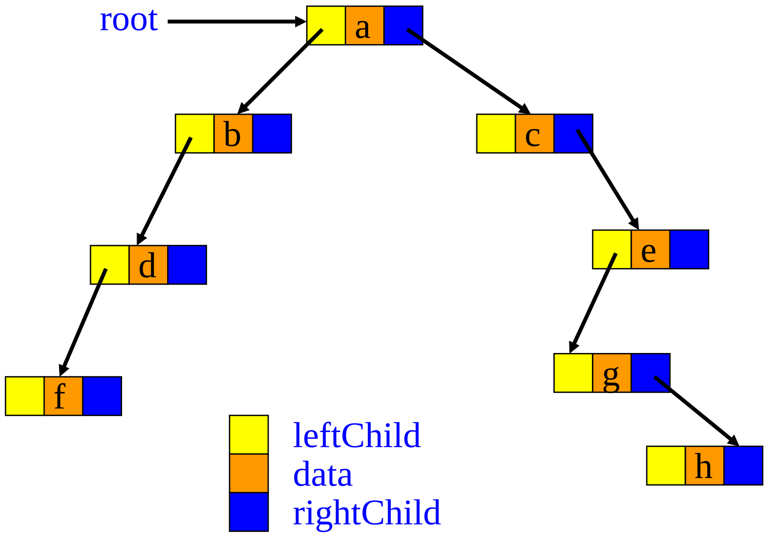
Linked Representation

- Each binary tree node is represented as an object whose data type is TreeNode.
- The space required by an n node binary tree is n * (space required by one node).

The Struct binaryTreeNode

```
template <class T>
class TreeNode
{
   T data;
   TreeNode<T> *leftChild,
               *rightChild;
   TreeNode()
      {leftChild = rightChild = NULL;}
   // other constructors come here
};
```

Linked Representation Example



🟨 leftChild
🟧 data
🟦 rightChild

Homework

- Sec. 5.2 Exercise 3 @P258