Evaluation of Expressions

Arithmetic Expressions

- (a + b) * (c + d) + e – f/g*h + 3.25
- Expressions comprise three kinds of entities.
  - Operators (+, -, /, *).
  - Operands (a, b, c, d, e, f, g, h, 3.25, (a + b), (c + d), etc.).
  - Delimiters ((,)).

Operator Degree

- Number of operands that the operator requires.
- Binary operator requires two operands.

  a + b

  c / d

  e - f
- Unary operator requires one operand.

  \+ g

  \- h

Infix Form

- Normal way to write an expression.
- Binary operators come in between their left and right operands.

  a * b

  a + b * c

  a * b / c

  (a + b) * (c + d) + e – f/g*h + 3.25

Operator Priorities

- How do you figure out the operands of an operator?
  - a + b * c
  - a * b + c / d
- This is done by assigning operator priorities.
  - priority(*) = priority(/) > priority(+) = priority(-)
- When an operand lies between two operators, the operand associates with the operator that has higher priority.

Evaluation Expression in C++

- When evaluating operations of the same priorities, it follows the direction from left to right.
- C++ treats
  - Nonzero as true
  - zero as false
  - !3&&5 +1 →0

Priority	Operator
1	Unary minus, !
2	*, /, %
3	+, -
4	<, <=, >=, >
5	== (equal), !=
6	&& (and)
7	|| (or)

In Class Exercise

- x=6, y=5
- 10+x*5/y+1
- (x>=5)&&y<10
- !x>10+!y

Tie Breaker

- When an operand lies between two operators that have the same priority, the operand associates with the operator on the left.
  - a + b - c
  - a * b / c / d

Delimiters

- Subexpression within delimiters is treated as a single operand, independent from the remainder of the expression.
  - (a + b) * (c – d) / (e – f)

Infix Expression Is Hard To Parse

- Need operator priorities, tie breaker, and delimiters.
- This makes computer evaluation more difficult than is necessary.
- Postfix and prefix expression forms do not rely on operator priorities, a tie breaker, or delimiters.
- So it is easier for a computer to evaluate expressions that are in these forms.

Postfix Form

- The postfix form of a variable or constant is the same as its infix form.
  - a, b, 3.25
- The relative order of operands is the same in infix and postfix forms.
- Operators come immediately after the postfix form of their operands.
  - Infix = a + b
  - Postfix = ab+

Postfix Examples

- Infix = a + b * c
  - a b c * +
- Infix = a * b + c
  - a b * c +
- Infix = (a + b) * (c – d) / (e + f)
  - a b + c d - * e f + /

Unary Operators

- Replace with new symbols.
  - + a => a @
  - + a + b => a @ b +
  - - a => a ?
  - - a-b => a ? b -

Postfix Notation

Expressions are converted into Postfix notation before compiler can accept and process them.

X = A / B – C + D * E – A * C

Infix => A / B – C + D * E – A * C (Operators come in-between operands)

Postfix => A B / C – D E * + A C * - (Operators come after operands)

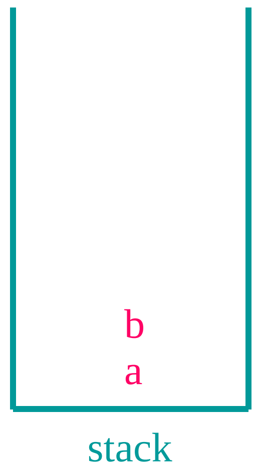
Operation	Postfix
T_1 = A / B	T_1 C – D E * + A C * -
T_2 = T_1 - C	T_2 D E * + A C * -
T_3 = D * E	T_2 T_3 + A C * -
T_4 = T_2 + T_3	T_4 A C * -
T_5 = A * C	T_4 T_5 -
T_6 = T_4 - T_5	T_6

Postfix Evaluation

- Scan postfix expression from left to right pushing operands on to a stack.
- When an operator is encountered, pop as many operands as this operator needs; evaluate the operator; push the result on to the stack.
- This works because, in postfix, operators come immediately after their operands.

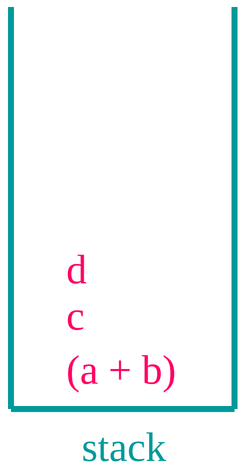
Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /



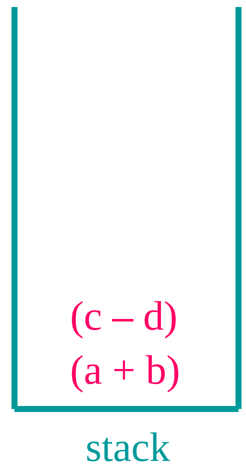
Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
- a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /



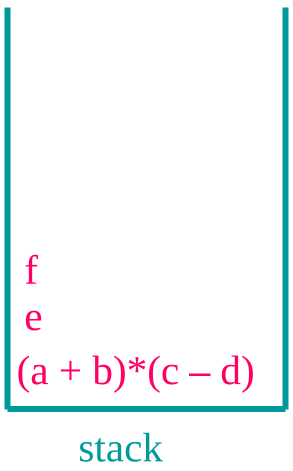
Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
  - a b + c d - * e f + /



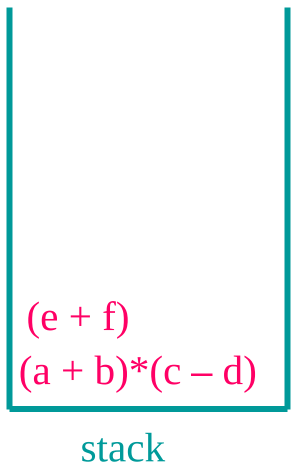
Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /



Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /
  - a b + c d - * e f + /



Infix to Postfix

- The order of the operands in both form is the same.
- An algorithm for producing postfix from infix:
  1. Fully parenthesize the expression.
  2. Move all operators so that they replace their corresponding right parentheses.
  3. Delete all parentheses.

Infix to Postfix

- For example: A/B-C+D*E-A*C
  1. Fully parenthesize the expression.
     ((((A/B)-C)+(D*E))-(A*C))
  2. Move all operators so that they replace their corresponding right parentheses.
     ((((AB/)C-)(DE*)+)(AC*)-)
  3. Delete all parentheses.
     AB/C-DE*+AC*-

In Class Exercise

- Write the postfix form:
  A&&B+C*D

Infix to Postfix

- We scan an expression for the first time, we can form the postfix by immediately passing any operands to the output.
- For example: A+B*C
  => ABC*+

Next token	Stack	Output
None	Empty	None
A	Empty	A
+	+	A
B	+	AB
*	+*	AB
C	+*	ABC

Since * has higher priority, we should stack *.

Infix to Postfix

- Example: A*(B+C)/D => ABC+*D/
- When we get ')', we want to unstack down to the corresponding '(' and then delete the left and right parentheses.

Next token	Stack	Output
None	Empty	None
A	Empty	A
*	*	A
(	*(	A
B	*(	AB
+	*(+	AB
C	*(+	ABC
)	*	ABC+
/	/	ABC+*
D	/	ABC+*D
Done	Empty	ABC+*D/

Infix to Postfix

- These examples motivate a priority-based scheme for stacking and unstacking operators.
- When the left parenthesis '(' is not in the stack, it behaves as an operator with high priority.
- whereas once '(' gets in, it behaves as one with low priority (no operator other than the matching right parenthesis should cause it to get unstacked)
- Two priorities for operators: isp (in-stack priority) and icp (in-coming priority)
- The isp and icp of all operators in Figure 3.15 in p 160 remain unchanged.
- We assume that isp('(') = 8 (the lowest), icp('(') = 0 (the highest), and isp('#') = 8 (# → the last token)

Infix to Postfix

- Result rule of priorities:
  - Operators are taken out of the stack as long as their isp is numerically less than or equal to the icp of the new operator.

Analysis of Postfix

- The function makes only a left-to-right pass across the input.
- The complexity of Postfix is Θ(n), where n is the number of tokens in the expression.
  - The time spent on each operands is O(1).
  - Each operator is stacked and unstacked at most once.
  - Hence, the time spent on each operator is also O(1)

Prefix Form

- The prefix form of a variable or constant is the same as its infix form.
  a, b, 3.25
- The relative order of operands is the same in infix and prefix forms.
- Operators come immediately before the prefix form of their operands.
  Infix = a + b
  Postfix = ab+
  Prefix = +ab

Prefix Examples

- Infix = a + b * c
  + a * b c
- Infix = a * b + c
  + * a b c
- Infix = (a + b) * (c – d) / (e + f)
  / * + a b - c d + e f

Prefix Notation

Expressions are converted into Prefix notation before compiler can accept and process them.

X = A / B – C + D * E – A * C

Infix => A / B – C + D * E – A * C (Operators come in-between operands)
Prefix => - + - / A B C * D E * A C (Operators come before operands)

Operation	Prefix
T_1 = A * C	A / B – C + D * E – T_1
T_2 = D * E	A / B – C + T_2 – T_1
T_3 = A / B	T_3 – C + T_2 – T_1
T_4 = T_3 – C	T_4 + T_2 – T_1
T_5 = T_4 + T_2	T_5 – T_1
T_6 = T_5 – T_1	T_6

Prefix Evaluation

- Scan prefix expression from right to left pushing operands on to a stack.
- When an operator is encountered, pop as many operands as this operator needs; evaluate the operator; push the result on to the stack.
- This works because, in prefix, operators come immediately before their operands.

Prefix Evaluation

- (a + b) * (c – d) / (e + f)
- / * + a b - c d + e f

- / * + a b - c d + e f
- / * + a b - c d + e f
- / * + a b - c d + e f

e
f

stack

Prefix Evaluation

- (a + b) * (c – d) / (e + f)
- / * + a b - c d + e f

- / * + a b - c d + e f
- / * + a b - c d + e f
- / * + a b - c d + e f

c
d
(e + f)

stack

Prefix Evaluation

- (a + b) * (c – d) / (e + f)
- / * + a b - c d + e f

- / * + a b - c d + e f
- / * + a b - c d + e f
- / * + a b - c d + e f

a
b
(c – d)
(e + f)

stack

Prefix Evaluation

- (a + b) * (c – d) / (e + f)
- / * + a b - c d + e f

- / * + a b - c d + e f

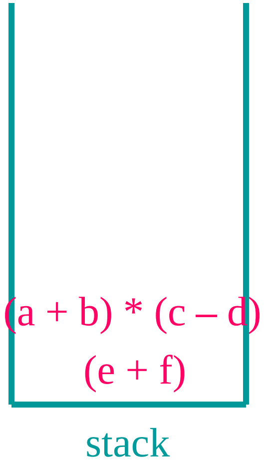
(a + b)
(c – d)
(e + f)

stack

Prefix Evaluation

- (a + b) * (c – d) / (e + f)
- / * + a b - c d + e f

- / * + a b - c d + e f



Infix to Prefix

- The order of the operands in both form is the same.
- An algorithm for producing prefix from infix:
  1. Fully parenthesize the expression.
  2. Move all operators so that they replace their corresponding left parentheses.
  3. Delete all parentheses.

Infix to Prefix

- For example: A/B-C+D*E-A*C
  1. Fully parenthesize the expression.
     ((((A/B)-C)+(D*E))-(A*C))
  2. Move all operators so that they replace their corresponding left parentheses.
     (-(+(-(/AB)C)(*DE))(*AC))
  3. Delete all parentheses.
     -+-/ABC*DE*AC

In Class Exercise

- Write the prefix form:
  A&&B+C*D

Infix to Prefix

- We reverse an expression at first
- Create empty reversed prefix String by passing any operands to the output.
- we can form the prefix by immediately reverse again the reversed prefix String.
- For example: A+B*C
  reverse: C * B+A
  => +A*BC

Next token	Stack	Reverse S
None	Empty	None
C	Empty	C
*	*	C
B	*	CB
+	+	CB*
A	+	CB*A
Done	Empty	CB*A+

Since * has higher priority, we should pop *, then push + .

Infix to Prefix

- Example: A*(B+C)*D
  reverse: D *)C+B(*A
  => **A+BCD
- When we get '(', we want to unstack down to the corresponding ')' and then delete the left and right parentheses.

Next token	Stack	Reverse S
None	Empty	None
D	Empty	D
*	*	D
)	*)	D
C	*)	DC
+	*)+	DC
B	*)+	DCB
(	*	DCB+
*	**	DCB+
A	**	DCB+A
Done	Empty	DCB+A**

don't pop *

Infix to Prefix

- These examples motivate a priority-based scheme for stacking and unstacking operators.
- When the right parenthesis ')' is not in the stack, it behaves as an operator with high priority.
- whereas once ')' gets in, it behaves as one with low priority (no operator other than the matching left parenthesis should cause it to get unstacked)
- Two priorities for operators: isp (in-stack priority) and icp (in-coming priority)
- The isp and icp of all operators in Figure 3.15 in p 160 remain unchanged.
- We assume that isp(')') = 8 (the lowest), icp(')') = 0 (the highest), and isp('#') = 8 (# → the last token)

Infix to Prefix

- Result rule of priorities:
  - Operators are taken out of the stack as long as their isp is numerically less than the icp of the new operator.
  - Not the same as Infix to Postfix

Analysis of Prefix

- The function makes only a left-to-right pass across the input (reversed prefix String).
- The complexity of Postfix is Θ(n), where n is the number of tokens in the expression.
  - The time spent on each operands is O(1).
  - Each operator is stacked and unstacked at most once.
  - Hence, the time spent on each operator is also O(1)

Homework

- Sec. 3.7 Exercise 3 (Page 166)
  - Convert infix expressions to prefix expressions